2021 Notes; Dated list of changes

Updates for 3/4/21

Page 259 Added Exer. 6.2.64 on the $P_\nu$ triviality of the $\sigma$-algebra of shift invariant events. Also, corrected and expanded Exer. 6.2.65 about $P_\nu$ triviality of the tail $\sigma$-algebra.

Page 277 Strengthened the statement of Prop. 7.1.9(c) by providing also its converse (and correspondingly adapting the proof of this part).

Page 277-278 Corrected a small error in Exmp. 7.1.11, by introducing the events $R^\infty_i$ and citing Exer. 6.2.64-6.2.65.

Page 278 Clarified the text of Exer. 7.1.15 (from HW8).

Updates for 3/1/21

Page 154 Revised text just before Theorem 4.1.2 to clarify that hereafter everything in Sec. 4.1 applies for all probability spaces and $\sigma$-algebras.

Page 219 Added to the remark following Thm. 5.5.11 a restatement of the Lebesgue decomposition (5.5.4) of $Q$ w.r.t. $P$ in terms of the Radon-Nikodym derivative $dP/dS$ for the dominating measure $S = \frac{1}{2}P + \frac{1}{2}Q$.

Pages 234-235 Revised the statement and proof of Prop. 6.1.16 so that it now applies to any $\mathcal{F}_n$-Markov chain and stopping time $\tau$, provided $h_n \in b\mathcal{F}^X$.

Page 276 Added in Example 7.1.5 the alternative definition of stationarity via f.d.d.

Pages 276, 278, 283 Clarified throughout Ch. 7 that stochastic process (or sequence) $\{X_n, n \geq 0\}$ being stationary, is equivalent to saying that its (joint) law $P_X$ is shift-invariant, eliminating from the notes all other combinations of notations.

Updates for 2/25/21

Page 63 Added Exercise 1.4.27 with construction of infinite product measure for independent ($S, S$)-valued random variables (thereby shifting by one items 1.4.27-1.4.46 of lecture notes).

Pages 224-226 Revised Definition 5.5.24, Lemma 5.5.25 Theorems 5.5.26, 5.5.28 and their proofs, so they all apply now for any $\mathcal{B}$-isomorphic $(S, S)$ (instead of only for $(\mathbb{R}, \mathcal{B})$).

Page 231 Added another identity in (6.1.3), extending the iterated integrals representation of $\mu_k(A)$ for any product set $A$ to apply for a product of $h_\ell \in bS$.

Pages 232 and 235 The new identity in (6.1.3) now justifies the middle equality in the display at the proof of Thm. 6.1.8 and the top display in proof of Prop. 6.1.16 (both of which previously lacking).

Page 233 Removed the redundant positive part in the definition of $X_n$ of Exer. 6.1.12(c).
Page 235 Added within the proof of the Markov property (6.1.8), also the measurability of $x \mapsto E_x[h]$, for any $h \in bS_c$.

Page 277 Fixed typo in the blue text: For stationary, irreducible, recurrent Markov chain on countable state space, the tail $\sigma$-field is non-trivial in the periodic case.

Updates for 2/18/21

Page 206 Statement of Exercise 5.3.39(b) modified, as it applies even if $P(\tau = \infty) > 0$. While we’ll post the solution for this more general statement, in HW6 the task is still to prove only the original statement.

Page 221 At start of proof of Prop. 5.5.13, replaced the reference to Exer. 4.1.8(b) (for independence), with a reference to Exer. 4.1.8(a) (for $M_n$ being of a product form).

Updates for 2/16/21

Page 203 Clarified the statement of Prop. 5.3.31.

Page 223 Added $L^1$ convergence as conclusion of Exer. 5.5.19.

Page 224 Minor changes in the text which follows (5.5.6).

Page 226 Specified the form of $f(\cdot)$ we take in proving Thm. 5.5.28.

Page 226 Clarified that in proof of Thm. 5.5.28 we utilize Prop. 1.4.21 (about independence), for the R.C.P.D. of $(\xi_1, \ldots, \xi_\ell)$ given $\mathcal{E}$, whose existence is a consequence of Exer. 4.4.4 and Prop. 1.4.27.

Updates for 2/14/21

Page 201 Typo fixed in the example preceding Prop. 5.3.22. Changed from $X_n = nI_{\{n \leq \tau\}}$ to $X_n = nI_{\{n < \tau\}}$.

Page 205 Added in Exer. 5.3.35 the (missing) assumption of the MG $(M_n, F_n)$ being $L^2$.

Updates for 2/11/21

Page 217 Change min to inf in the formal definition of the time of extinction $T$.

Page 219 Minor text changes within the derivation of $M_n = (2 - V_n)/V_n$ (while proving Thm. 5.5.11(c)).

Page 220 Now explicitly stating that we’ve showed that $Q$-a.s. $M_n \to M_\infty$ (while proving Thm. 5.5.11(c)), as already cited within our proof of Prop. 5.5.13.

Pages 220 and 221 Expanded the texts which precede Prop. 5.5.13 and Exercise 5.5.15, to better highlight how the former is being employed in statistical (sequential) decision theory.

Page 222 Fixed a typo in the remark which follows Thm. 5.5.18 (should be there $n \leq 0$, not $n \geq 0$).
Updates for 2/8/21

Page 203 Generalized the statement and proof of Thm. 5.3.33(b).

Updates for 2/4/21

Page 205 New remark extends the scope of Prop. 5.3.34.

Updates for 2/2/21

Page 188 Added an intermediate step $XI_A \leq (X)_+I_A$, at the end of proof of Theorem 5.2.6.

Page 191 Re-wrote (5.2.4) to allow for a more direct proof that uses (5.2.1) for the non-negative subMG $(X_k)_+$.

Page 191 Fixed typo in the text preceding Exercise 5.2.14.

Page 201 Added explicit reference to the a.s. convergence $Y_n \rightarrow 0$ early in the proof of Prop. 5.3.22.

Updates for 1/31/21

Page 197 In Exercise 5.3.9(a) we want an example of a sub-MG $X_n$ such that $X_n^2$ is a sup-MG and also not a sub-MG. Added the latter requirement to text of Exercise 5.3.9(a).

Updates for 1/27/21

Page 154 Remark now clarifies in what sense allowing $\sigma$-algebra $G$ be more general than conditioning only on a random variable.

Page 154 For more intuition about (4.1.1), added a comment that it directly follows from requiring the trivial tower identity (4.2.1) and the take out what is known property (as in Prop. 4.2.10).

Page 155 Expanded the remark to clarify that Lebesgue decomposition provides a converse for Radon-Nykodim theorem.

Page 171 Added a remark following Prop. 4.4.1 with an interpretation of $\hat{g}(z)$ as the limit of conditioning upon intervals of shrinking length.

Page 172 Added a remark on the difference between the probability measure $P(\cdot|B)$ and the R.C.P. given $\sigma(B)$.

Page 177 Clarified that our use of “information” in the context of filtration, does not imply a connection with mutual-information (as defined in the field of information theory).

Updates for 1/25/21

Page 180 Fixed typo in Example 5.1.10. Assuming only that $Y_k \geq 0$ (instead of strictly positive), we only have that $\mathcal{F}_n^M \subseteq \mathcal{F}_n^Y$.

Pages 180, 188, 203, 209 Changed min to inf in Exercise 5.1.13, Theorem 5.2.6, within the proof of Theorem 5.3.33(a) and in Exercise 5.4.7, where the values may be infinity (i.e. no achieving $k$, or $n$, respectively).
Page 207 Fixed typo in second line of proof of Theorem 5.4.1, ($V_n = 0$ instead of $V_n$).

**Updates for 1/11/21**

Page 154 Remark expanded, to clarify that all properties of the conditional expectation hold only in the a.s. sense (even when this qualifier is omitted).

**Updates for 1/9/21**

Page 156 Clarified that in part (b) of Exercise 4.1.8, $\mu$ and $\nu$ are both product probability measures, as in part (a).

**Updates for 12/15/20**

Page 221 Corrected typo in Exercise 5.5.16(b). The variable $Y$ should be integrable with respect to $Q_n$ or equivalently, $Q_m$ (not as before, integrable with respect to $P$).