2021 Notes; Dated list of changes

Updates for 2/2/21

Page 188 Added an intermediate step $XI_A \leq (X)_+I_A$, at the end of proof of Theorem 5.2.6.

Page 191 Re-wrote (5.2.4) to allow for a more direct proof that uses (5.2.1) for the non-negative subMG $(X_k)_+$.

Page 191 Fixed typo in the text preceding Exercise 5.2.14.

Page 201 Added explicit reference to the a.s. convergence $Y_n \to 0$ early in the proof of Prop. 5.3.22.

Updates for 1/31/21

Page 197 In Exercise 5.3.9(a) we want an example of a sub-MG $X_n$ such that $X_n^2$ is a sup-MG and also not a sub-MG. Added the latter requirement to text of Exercise 5.3.9(a).

Updates for 1/27/21

Page 154 Remark now clarifies in what sense allowing σ-algebra $\mathcal{G}$ be more general than conditioning only on a random variable.

Page 154 For more intuition about (4.1.1), added a comment that it directly follows from requiring the trivial tower identity (4.2.1) and the take out what is known property (as in Prop. 4.2.10).

Page 155 Expanded the remark to clarify that Lebesgue decomposition provides a converse for Radon-Nykodim theorem.

Page 171 Added a remark following Prop. 4.4.1 with an interpretation of $\hat{g}(z)$ as the limit of conditioning upon intervals of shrinking length.

Page 172 Added a remark on the difference between the probability measure $P(\cdot | B)$ and the R.C.P. given $\sigma(B)$.

Page 177 Clarified that our use of “information” in the context of filtration, does not imply a connection with mutual-information (as defined in the field of information theory).

Updates for 1/25/21

Page 180 Fixed typo in Example 5.1.10. Assuming only that $Y_k \geq 0$ (instead of strictly positive), we only have that $\mathcal{F}_n^M \subseteq \mathcal{F}_n^Y$.

Pages 180, 188, 203, 209 Changed min to inf in Exercise 5.1.13, Theorem 5.2.6, within the proof of Theorem 5.3.33(a) and in Exercise 5.4.7, where the values may be infinity (i.e. no achieving $k$, or $n$, respectively).

Page 207 Fixed typo in second line of proof of Theorem 5.4.1, ($V_n = 0$ instead of $V_n$).

Updates for 1/11/21

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Page 154 Remark expanded, to clarify that all properties of the conditional expectation hold only in the a.s. sense (even when this qualifier is omitted).

Updates for 1/9/21

Page 156 Clarified that in part (b) of Exercise 4.1.8, $\mu$ and $\nu$ are both product probability measures, as in part (a).

Updates for 12/15/20

Page 221 Corrected typo in Exercise 5.5.16(b). The variable $Y$ should be integrable with respect to $Q_n$ or equivalently, $Q_m$ (not as before, integrable with respect to $P$).