1. Using ade4

(a) The ade4 package was installed in a local directory `/location of ade4/` and call it with the command
```r
library(ade4, lib.loc='/location of ade4/')
```

(b) Load the skulls data, perform a discriminant analysis with ade4 using `discrim` and using `lda` from MASS, compare the results.

(c) What percentage of the between group inertia is explained by the first discriminating function.

(d) What is the cross validated estimate of the probability of being well classified?

2. The EM algorithm

(a) Write your own algorithm for solving the following EM problem of the mixture of 2 normal samples. For instance, if the observations $x_i$’s are independently distributed as $\theta g(x) + (1 - \theta)h(x)$, where $g$ and $h$ are known probability densities. Our goal is to find MLE of $\theta$ given the observations $(x_1, x_2, ..., x_n)$. Derive an EM algorithm using the following steps:

(a) Introduce the binary categorical variables $Z_1, Z_2, ..., Z_n$, where $z_i$ indicates from which density $x_i$ has been drawn. (That is, if $Z_i = 1$ then $x_i \sim g(x)$ and if $z_i = 0$ then $x_i \sim h(x)$).

(b) Show that the complete data likelihood can be written as:

$$
\prod_{i=1}^{n} (z_i g(x_i) + (1 - z_i)h(x_i)) \theta^{z_i}(1 - \theta)^{(1 - z_i)}.
$$

(c) Show that the conditional expectation of $Z_i$ is:

$$E[Z_i | \theta_0, x_i] = \frac{\theta_0 g(x_i)}{\theta_0 g(x_i) + (1 - \theta_0)h(x_i)}.$$

(d) Write down the EM algorithm for estimation of $\theta$.

(e) Implement this algorithm for $g$ being $N(0, 1)$ and $h$ being normal $N(4, 1)$, and the following observations:

$(0.95, 0.23, 4.45, 0.60, 4.44, 0.48, -0.42)$.

(b) Find a package that performs the mixture decomposition algorithm in the normal case and test your answers are correct, explain how you found the package.