Q. 1) (a) Let $X$ be the average number of monthly crashes, then $X$ could be a Poisson random variable with mean 3.5 or $\lambda = EX = 3.5$. So

$$P\{X = i\} = e^{-\lambda} \frac{\lambda^i}{i!}, \lambda = 3.5, i = 0, 1, 2, \ldots$$

Therefore

$$P\{X \geq 2\} = 1 - P\{X = 0\} - P\{X = 1\} = 1 - e^{-3.5} - 3.5e^{-3.5} = 1 - 4.5e^{-3.5}$$

(b) $P\{X \leq 1\} = 1 - P\{X \geq 2\} = 4.5e^{-3.5}$

Q. 2) If $X = k$, then there are $k$ tails and 9 heads in first $k + 9$ trials, and the $(k + 10)$th trial is head. Notice that there are $\binom{k+9}{9}$ ways to occur in the first $k + 9$ trials. Hence

$$P(X = k) = \binom{k+9}{9} \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^9 = \binom{k+9}{9} \left(\frac{1}{2}\right)^{k+10}$$

Q. 3) The integration by parts formula $\int udv = uv - \int vdu$ with $u = -x/2b$, $v = e^{-bx^2}, dv = -2bxe^{-bx^2}$ yields that

$$\int_0^\infty x^2 e^{-bx^2} dx = \left[ -x e^{-bx^2} \right]_0^\infty + \frac{1}{2b} \int_0^\infty e^{-bx^2} dx$$

$$= \frac{1}{(2b)^{3/2}} \int_0^\infty e^{-y^2/2} dy, \quad y = x\sqrt{2b}$$

$$= \frac{\sqrt{\pi}}{4b^{3/2}}$$

where the above uses that $\int_0^\infty e^{-y^2/2} dy = 1/2$.

Hence, $\int_0^\infty ax^2 e^{-bx^2} dx = 1$ yields that $a = \frac{4b^{3/2}}{\sqrt{\pi}}$

Q. 4) (a) $P\{X > a\} = \int_a^\infty \frac{1}{2\pi} e^{-\frac{x^2}{2}} dx = -e^{-\frac{a^2}{2}} \bigg|_a^\infty = e^{-\frac{a^2}{2}}$, so

$$P\{X > 2\} = e^{-4}$$

Therefore

$$P\{X \leq 1\} = 1 - P\{X \geq 2\} = 4.5e^{-3.5}$$

Hence, $\int_0^\infty ax^2 e^{-bx^2} dx = 1$ yields that $a = \frac{4b^{3/2}}{\sqrt{\pi}}$
Q. 5) (a) When n large and k is not too big, the drawing can be regarded as with replacement. That means two balls are withdrawn at a time and they are put back. Then each drawing is an independent trial which is success when two drawn balls have the same number.

(b) Each independent trial successes with probability \( n \choose 2 \frac{1}{n} \). So \( P\{M_k = 0\} = P\{\text{first k selections are all failed}\} = (1 - \frac{1}{2n-1})^k \)

(c) \( \{T > an\} = \{M_{\lfloor an\rfloor} = 0\} \), where \([x]\) is the largest integer not bigger than \(x\).

(d)

\[
\lim_{n \to \infty} P(T > an) = \lim_{n \to \infty} P(M_{\lfloor an\rfloor} = 0)
= \lim_{n \to \infty} (1 - \frac{1}{2n-1})^{\lfloor an\rfloor}, \text{ by (b)}
= e^{-\lim_{n \to \infty} \frac{\lfloor an\rfloor}{2n-1}} = e^{-a/2}
\]