Statistics 116 - Fall 2004
Theory of Probability
Assignment # 3
Due Friday, October 15
SHOW (AND BRIEFLY EXPLAIN) ALL OF
YOUR WORK

Q. 1) (Ross #2.11 - Theoretical) If \( P(E) = 0.9 \) and \( P(F) = 0.8 \), show that \( P(E \cap F) \geq 0.7 \). In general, prove Bonferroni’s inequality, namely
\[
P(E \cap F) \geq P(E) + P(F) - 1.
\]

Q. 2) (Ross #3.4) What is the probability that at least one of a pair of fair dice lands on 6, given that the sum of the dice is \( i \), for \( i = 2, 3, \ldots, 12 \)?

Q. 3) (Ross #3.24) Suppose that 5 percent of men and .25 percent of women are colorblind. A colorblind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females. What if the population consisted of twice as many males as females?

Q. 4) (Ross #3.6 - Theoretical) Prove that if \( E_1, \ldots, E_n \) are independent events, then
\[
P(E_1 \cup E_2 \cup \cdots \cup E_n) = 1 - \prod_{i=1}^{n}[1 - P(E_i)].
\]

Q. 5) (Ross #3.12 - Theoretical) The probability of getting a head on a single toss of a coin is \( p \). Consider that \( A \) starts and continues to flip the coin until a tail shows up, at which point \( B \) starts flipping. Then \( B \) continues to flip until a tail comes up, at which point \( A \) takes over, and so on. Let \( P_{n,m} \) denote the probability that \( A \) accumulates a total of \( n \) heads before \( B \) accumulates \( m \). Show that
\[
P_{n,m} = p \cdot P_{n-1,m} + (1 - p) \cdot (1 - P_{m,n}).
\]