Similarity between Brain and Perceptual Representations of Language

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Research Program

1. Cryptologists of language in the brain.
2. Structural isomorphism should exist.
   Why? Needed to easily understand each other’s speech.
3. Usually in psychology and neuroscience, too hard to find a structural isomorphism in any rigorous sense.
Research Program

4. But ordinal similarity long studied in psychology.
5. This weaker concept of similarity is appropriate for current weak understanding of brain computations focused on language.
### Miller-Nicely Confusion Matrix

<table>
<thead>
<tr>
<th>Sound</th>
<th>$p^-$</th>
<th>$t^-$</th>
<th>$b^-$</th>
<th>$g^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p^+$</td>
<td>$t^+$</td>
<td>$b^+$</td>
<td>$g^+$</td>
</tr>
<tr>
<td>$p^-$</td>
<td>51</td>
<td>53</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$t^-$</td>
<td>64</td>
<td>57</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$b^-$</td>
<td>4</td>
<td>2</td>
<td>60</td>
<td>18</td>
</tr>
<tr>
<td>$g^-$</td>
<td>3</td>
<td>1</td>
<td>20</td>
<td>29</td>
</tr>
</tbody>
</table>
## Brain Data Confusion Matrix

<table>
<thead>
<tr>
<th>Test Samples</th>
<th>Prototypes</th>
<th>( p^+ )</th>
<th>( t^+ )</th>
<th>( b^+ )</th>
<th>( g^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p^- )</td>
<td>36</td>
<td>38</td>
<td>12</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>( t^- )</td>
<td>34</td>
<td>50</td>
<td>3</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>( b^- )</td>
<td>14</td>
<td>31</td>
<td>42</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>( g^- )</td>
<td>10</td>
<td>33</td>
<td>15</td>
<td>42</td>
<td></td>
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</table>
First Steps of Analysis

1. Normalize each row of a confusion matrix to probabilities summing to 1.

2. The conditional probability

\[ o_j^+ \mid o_i^- = \text{Prob}(\text{Prototype } j \mid \text{Test sample } i) \]

= the measure of similarity of prototype \( j \) and test sample \( i \)

For example,

\[ p(t^+ \mid t^-) = .50 \]
\[ p(t^+ \mid b) = .03 \]
Semiorders and the Invariants of Intersections, which are Partial Orders

- Conditional probability density

\[ p(o_j^+ | o_i^-) \geq 0 \text{ and } \sum_{j=1}^{N} p(o_j^+ | o_i^-) = 1 \]

- Estimate from confusion matrix

\[ \hat{p}(o_j^+ | o_i^-) = \frac{m_{ij}}{\sum_j m_{ij}} \]

- Ordinal similarity

\[ o_j^+ | o_i^- \succ o_{j'}^+ | o_{i'}^- \iff p(o_j^+ | o_i^-) > p(o_{j'}^+ | o_{i'}^-) \]
Definitions

• Let $R$ be irreflexive on $A$. Then $R$ is an **interval order** on $A$ iff for every $a, b, c$ and $d$ in $A$, if $aRb$ and $cRd$, then $aRd$ or $cRb$.

• $R$ is **strongly transitive** on $A$ iff for every $a, b, c$ and $d$ on $A$, if $aRb$ and $bRc$ then either $dRc$ or $aRd$.

• $R$ is a **semiorder** on $A$ iff $R$ is irreflexive, strongly transitive and an interval order on $A$. 

Numerical Threshold

• Any finite semiorder \((A, \prec)\) has a numerical representation \(\varphi\) with a positive threshold such that for \(a\) and \(b\) in \(A\)

\[
\varphi(a) + 1 < \varphi(b) \text{ iff } a \prec b
\]

\[
o_j^+ | o_i^- \approx o_j^+ | o_i^- \iff |p(o_j^+ | o_i^-) - p(o_j^+ | o_i^-)| \leq 0.02
\]

\[
o_j^+ | o_i^- > o_j^+ | o_i^- \iff p(o_j^+ | o_i^-) > p(o_j^+ | o_i^-) + 0.02
\]
Conditional Probabilities of 4 consonants (Miller-Nicely)

<table>
<thead>
<tr>
<th></th>
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<th>t</th>
<th>b</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
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<td>.49</td>
<td>.02</td>
<td>.02</td>
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<tr>
<td>t</td>
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<td>.46</td>
<td>.02</td>
<td>.01</td>
</tr>
<tr>
<td>b</td>
<td>.05</td>
<td>.02</td>
<td>.71</td>
<td>.21</td>
</tr>
<tr>
<td>g</td>
<td>.06</td>
<td>.02</td>
<td>.38</td>
<td>.55</td>
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</table>
Methods of Analysis of Brain Data

1. EEG recordings.
2. Fourier analysis.
3. Linear Discriminant Model (LDC)
4. LDC with regularization.
6. Almost everywhere, ICA cleaning of data prior to analysis.
### Exp I: Conditional Probabilities (EEG-LDC for S7)

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Semiorder Graphs of Similarities

Miller-Nicely

S7 Brain

Diagram showing the relationships between different nodes in a graph for Miller-Nicely and S7 Brain.
Invariants of Intersection of M-N and EEG-LDC of S7

- Robust discrimination of voicing: $p$ not similar to $b$.
- Strong attraction of $p$ and $t$.
- Strong similarity of prototype and test samples of each phoneme.
Compare the Trees

**Miller-Nicely**

```
+g
-g
+b
-b
+p
-t
+t
-p
```

**S7 Brain**

```
+g
-g
+b
-b
+t
-t
+p
-p
```

Brain representation has more expected similarities than perceptual representation
## Conditional Probabilities (EEG-LT) of S5

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<td>b</td>
<td>.07</td>
<td>.11</td>
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<td>.14</td>
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<tr>
<td>g</td>
<td>.11</td>
<td>.14</td>
<td>.16</td>
<td>.59</td>
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Semiorder Graphs of Similarities

**Miller-Nicely**

- **b|b**
- **g|g**
- **p|t**
- **t|p**
- **p|p**
- **t|t**
- **b|g**
- **g|b**
- **p|b**
- **p|g**
- **t|b**
- **t|g**
- **b|p**
- **g|t**

**S5 Brain**

- **b|b**
- **t|t**
- **g|g**
- **p|p**
- **b|p**
- **t|p**
- **g|p**
- **b|g**
- **p|t**
- **t|g**
- **g|b**
- **b|t**
- **p|g**
- **t|b**
- **g|t**
- **p|b**
Invariants of Intersection of M-N and EEG-LT of S5

- Strong similarity of prototypes and matching samples.
- $p$ and $t$ still close.
- $b$ and $t$ separated (differ in voicing and place of articulation).
Compare the Trees

**Miller-Nicely**

- +p
  - -t
    - +t
    - +p
  - -t
    - +t
    - +p
- -p

**S5 Brain**

- +t
  - -t
    - +g
    - -g
  - -g
    - +b
    - -b
  - +b
    - -b
    - +p
    - -p
Figure 3: Semiorder graph of similarities for Experiment II VIS LDC. The semiorder was constructed from the simple order of the conditional probabilities using a threshold $\epsilon = 0.02$. 
Figure 4: Semiorder graph of similarities of auditorily generated brain waves for Experiment II Aud LDC with a threshold $\epsilon = 0.02$. 
Figure 5: Semiorder graph of similarities for auditory words using a distance measure with a threshold $\epsilon = 0.02$. 
Figure 6: Intersection of the two semiorders of Experiment II using similarities of visually and auditorily generated brain waves (Figures 3 & 4).
Figure 7: Intersection of the two similarity semiorders of Experiment II of brain waves generated by auditory words (Figure 4) and the auditory words themselves (Figure 5).
Figure 8: Similarity semiorder for the names of three cities without considering position, (a) visually generated brain waves, (b) auditorily generated brain waves, and (c) the auditory words
Figure 9: Pairwise intersections of the three semiorders of Figure 8, (a) auditorily vs visually generated brain waves, (b) auditorily generated brain waves vs the auditory words, and (c) visually generated brain waves vs the auditory words.
Figure 10: Semiorder of similarities for visually generated brain waves for Experiment III+IV, Model LDC with a threshold of $\epsilon = 0.02$. 
Figure 11: Semiorder of similarities for visually generated brain waves for Experiment III+IV, Model LDC-TR with a threshold $\epsilon = 0.02$. 
Figure 12: Intersection of the two similarity semiorders for Experiment III+IV LDC (Figure 10) vs LDC-TR (Figure 11).