8.3 Lecture 9 Monday 02/05/01

8.4 Fitting Gamma densities

From the Illinois data,the fit was made using the estimates from the sample:

$$\bar{X} = .224 and \qquad \hat{\sigma^2} = .1332 \Longrightarrow \hat{\alpha} = .35 and \qquad \hat{\lambda} = 1.68$$

Here is how to do this with Matlab.

8.4.1 Matlab uses

```
>> lookfor gamma
GAMMA  Gamma function.
GAMMAINC Incomplete gamma function.
GAMMALN Logarithm of gamma function.
BLSGAMMA Black-Scholes sensitivity to underlying delta change.
CMGAMDEF Default gamma correction table.
CMGAMMA Gamma correct colormap.
ruqvsol.m: % function [q,gammaopt] = ruqvsol(r,u,v)
DHINFOPT Discrete H-Infinity control synthesis via Gamma iteration.
HINFOPT H-Infinity control synthesis via Gamma iteration.
GAMCDF Gamma cumulative distribution function.
GAMFIT Parameter estimates and confidence intervals for gamma distributed data.
GAMINV Inverse of the gamma cumulative distribution function (cdf).
GAMLIKE Negative gamma log-likelihood function.
GAMPDF Gamma probability density function.
GAMRND Random matrices from gamma distribution.
GAMSTAT Mean and variance for the gamma distribution.
HINFOPT H-Infinity control synthesis via Gamma iteration.
GAMMA  Symbolic gamma function.
```

Important ones here: GAMFIT (We'll do later). GAMPDF GAMCDF GAMRND

```
 GAMRND Random matrices from gamma distribution.
    R = GAMRND(A,B) returns a matrix of random numbers chosen
    from the gamma distribution with parameters A and B.
```

In the file, replace the '*'s systematically with 'NaN'.

```
load('illrain.txt')
ISNAN True for Not-a-Number.
>> sum(~isnan(illrain(:,2)))
ans =
    48
>> mean(illrain(~isnan(illrain(:,2)),2))
ans =
    0.2749
>> storms60=illrain(~isnan(illrain(:,1)),1);
>> storms61=illrain(~isnan(illrain(:,2)),2);
>> storms62=illrain(~isnan(illrain(:,3)),3);
>> storms63=illrain(~isnan(illrain(:,4)),4);
>> storms64=illrain(~isnan(illrain(:,5)),5);
allstorms=[ storms60; storms61;storms62; storms63;storms64];
>> mean(allstorms)
    0.2244
>> (226/227)*var(allstorms)
    0.1332
```

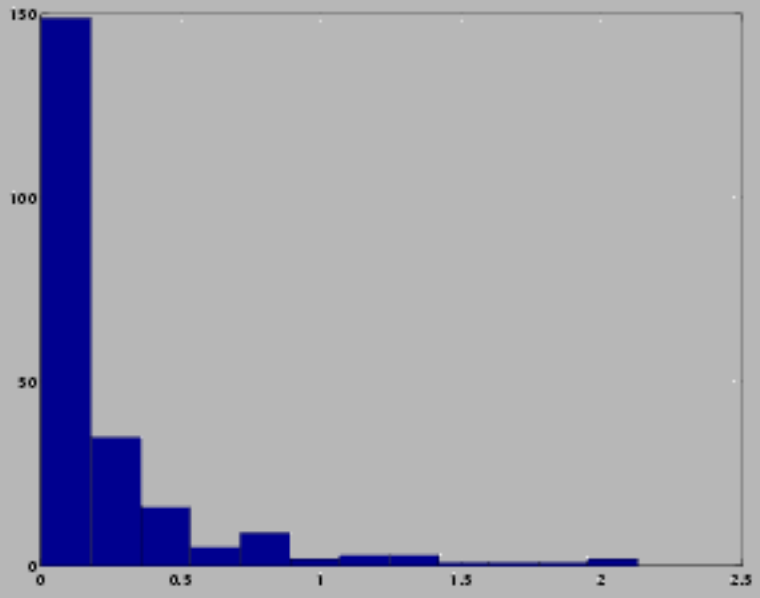

Figure 1: Histogram of all storms

```
>> lambdah=mean(allstorms)/(0.1332)
lambdah =
    1.6846
```

```
>> alphah=0.2244^2/(0.1332)
alphah =
    0.3780
%%%%Frequency Polygon computation and plot
>> [freqs,binc]=hist(allstorms,50);
>> plot(binc,freqs)
>>plot(binc,freqs/227)
>>pdf=gampdf(0.005:0.05:2.4,0.378,1/1.68);
>>hold on;
>> plot(0.005:0.05:2.4,pdf*10,'r-')
```

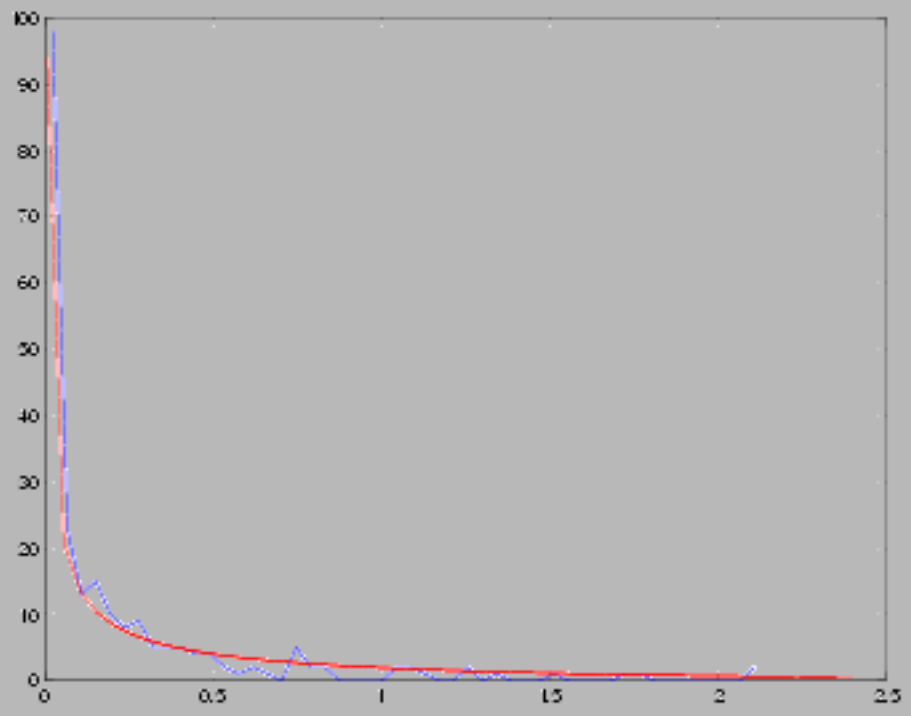

Figure 2: Frequency polygon

8.4.2 Sampling Distribution for the parameters:

What about the sampling distribution of $\hat{\alpha}$ and $\hat{\lambda}$?

For α large we could actually work it out. it would be approximately a ratio of a χ^2_1 and a χ^2_{n-1} could be close to a $F_{1,n-1}$. this has to be worked out and proved using the delta method. it is hard and clumsy.

We now use the parametric bootstrap which is inspired from the theoretical situation:

In a perfect world, if we were allowed a wish, and we wanted to get an idea about the estimates variability we construct many samples from teh original one, specified by its form and the two parameters:

$$\Gamma(\alpha_0, \lambda_0)$$

Suppose the true distribution IS $\Gamma(.378, 1.68)$. Make 1000 such samples, estimate the parameters by the method of moments, plot the sampling distribution.

```
-------------------------------------------------
function out=samplegam(alpha,lambda,n,S)

%out will contain 2 columns, one for each parameter
%and S rows, one for each simulation experiment
out=zeros(S,2);
%generate all the samples at once
%n is the sample size
samples=gamrnd(alpha,1/lambda, n,S);
for (j=1:S)
   %Estimate parameters from each sample in turn
   xbar=mean(samples(:,j));
   varh=((n-1)/n)*var(samples(:,j)) ;
   out(j,1)=xbar^2/varh
   out(j,2)=xbar/varh
end %for loop
-------------------------------------------------

>> out1=samplegam(alphah,lambdah,227,1000);
>> mean(out1(:,2))
    1.7841
>> mean(out1(:,1))
    0.3952
>> subplot(2,1,1)
>> hist(out1(:,1))
>> title('Sampling dist for alpha')
>> subplot(2,1,2)
>> hist(out1(:,2))
>> title('Sampling dist for lambda')
>> std(out1(:,1))
    0.0647
>> std(out1(:,2))
    0.3574
```

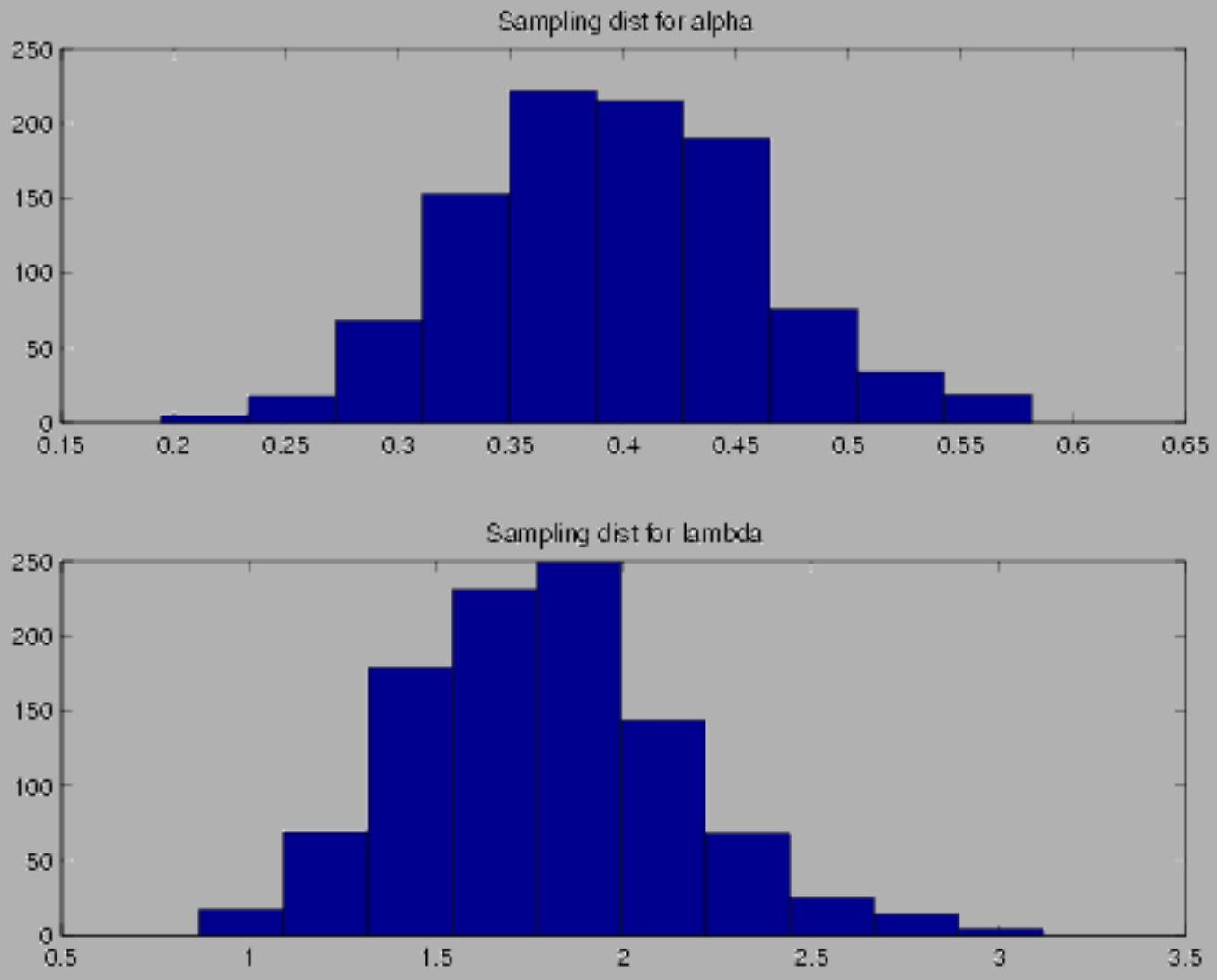

Figure 2: Sampling Distributions for both parameters

8.4.3 An angular distribution

We study the angle θ at which electrons are emitted in muon decay, its cosine has a density ($x = cos\theta$):

$$f(x|\alpha) = \frac{1 + \alpha x}{2}$$

α is the parameter we are interested in, it is related to polarization.

$$\mu = \int_{-1}^{1} x(\frac{1 + \alpha x}{2})dx = \left[\frac{x^2}{4} + \frac{\alpha x^3}{6}\right]_{-1}^{1} = \frac{\alpha}{3}$$

Thus the method of moments tells us to estimate α by $\hat{\alpha} = 2\bar{X}$. Summary of the method:

1. Write the moments as function of the parameters.

2. Write the parameters as a function of the moments.

3. Plug in the moment estimates computed from the sample in the functions.

3.4.4 Consistency

Under certain reasonable conditions, when the sample size increases the method of moments estimates (the constructed approach the true parameters more and more); the estimates are said to be **consistent**.

Definition:
Let $\hat{\theta}_n$ be an estimate of a parameter θ, (the n this n signifies it is based on a sample of size n), the estimate $\hat{\theta}_n$ is said to be consistent in probability if $\hat{\theta}_n$ converges in probability to θ when $n \longrightarrow \infty$;

$$\forall \epsilon, P(|\hat{\theta}_n - \theta| > \epsilon) \longrightarrow 0 \quad \text{when} \quad n \longrightarrow \infty$$

Weak Law of Large Numbers (WLLN,page 164 in Ross, and see pages 46-52, Lehmann) says that the $\bar{X}_n$ converges in probability to μ.

$$\forall \epsilon, P(|\bar{X}_n - \mu_X| > \epsilon) \longrightarrow 0 \qquad \text{as} \qquad n \longrightarrow \infty$$

If the functions relating moments and parameters are continuous functions the estimates converge to the parameters, this justifies the estimation of the standard errors we have seen using:

$$\sigma_{\hat{\theta}} = \frac{1}{\sqrt{n}}\sigma(\hat{\theta}_n) \text{ has been replaced by } \hat{\sigma}_{\hat{\theta}} = \frac{1}{\sqrt{n}}\sigma(\hat{\theta})$$