Homework Assignment Number 6:
Due Friday, March 16th, 2001

1. A dental study found that the mean difference in the number of new cavities after a trial period between independent samples of children who used respectively, Colgates MFP fluoride toothpaste and a stannous fluoride toothpaste was 2.41.

   The mean difference $\bar{D}$ is assumed to be Normal with mean θ and variance 1.25.

   (a) Find a 95% confidence interval for θ and calculate its probability if the researcher's prior is:
   a) Normal(4,4)
   b) Normal(1,16)

   (b) Find the probability of the null hypothesis $\theta \leq 0$ in each case.

2. The prior distribution with probability density

   $$g(p) \propto [p(1-p)]^{-1/2}, 0 < p < 1$$

   is known as Jeffrey's prior for the Bernouilli parameter p.

   (a) Find the constant of proportionnality.

   (b) Find the (unconditional) propobability of success, assuming Jeffrey's prior.

   (c) Suppose you have adopted Jeffrey's prior and observe a success. What is the new probability of success at the next trial? That is, given that one trial results in a success find the predictive distribution of p for the next trial.

3. In class I showed an example of using a mixture of two or three betas for the penny spinning problem.

   Show how in the details one can find the weights of the posterior distribution if the prior is one of the three following mixtures

   $$\begin{aligned} (\mathbf{i}) \quad & \beta(4,8) \\ (\mathbf{ii}) \quad & 0.5\beta(10,20) + 0.5\beta(20,10) \\ (\mathbf{i}) \quad & 0.5\beta(10,20) + 0.2\beta(15,15) + 0.3\beta(20,10) \end{aligned}$$

   and we observe 3 heads out of 10 trials.
   Hint: The posterior probability weights will be proportional to $\frac{1}{2}\frac{B(13,27)}{B(10,20)}$ and $\frac{1}{2}\frac{B(23,17)}{B(20,10)}$ for (ii), show why.

   Bonus(extra credit): Do the computation by simulation: here is the code for the beta mixtures. Here are the matlab commands to generate the prior and posterior distns:

```
 ps=(0.001:0.01:1);
 pr2=0.5*betapdf(ps,10,20)+0.5*betapdf(ps,20,10);
 pr3=0.5*betapdf(ps,10,20)+0.2*betapdf(ps,15,15)+0.3*betapdf(ps,20,10);
 po1=betapdf(ps,4,8);
po2=0.84*betapdf(ps,13,27)+0.16*betapdf(ps,23,17);
po3=0.77*betapdf(ps,13,27)+0.16*betapdf(ps,18,22)+0.07*betapdf(ps,23,17);
```