**Sketch of the Solution : HW2**

1. *Prob 7.18*
   Find $\alpha$ from the Normal table( Table 2 in appendix), satisfying $Z_{1-\alpha/2} = 1$. $\alpha = 2 \cdot 0.1587 = 0.3174$
   Therefore the the size of the confidence interval is 0.6826

2. *Prob 7.26*
   (a) The unbiased estimate of the standard error of this estimate is

$$s^2_{\widehat{p}} = \frac{\widehat{p}(1-\widehat{p})}{n-1}(1-\frac{n}{N}) = 0.0007293467$$

   Therefore, the 90% confidence interval is $\widehat{p} \pm Z_{1-0.05} \cdot s_{\widehat{p}} = 0.18 \pm 1.645 \cdot 0.02700642 = 0.18 \pm 0.04442556$.
   (b) Since $\widehat{p_1}$ and $\widehat{p_2}$ are independent, $Var(\widehat{d}) = Var(\widehat{p_1}) + Var(\widehat{p_2}) = \frac{\widehat{p_1}(1-\widehat{p_1})}{n-1}(1-\frac{n}{N}) + \frac{\widehat{p_2}(1-\widehat{p_2})}{n-1}(1-\frac{n}{N}) = 0.03^2 + 0.027^2 = 0.001629$
   (c) The standard error of $\widehat{d}$ is 0.04036. Therefore,

$$\begin{aligned} 99\ \%\ C.I &= -0.06 \pm 2.575 \cdot 0.04036 = (-0.16401 \quad 0.0440) \\ 95\ \%\ C.I &= -0.06 \pm 1.960 \cdot 0.04036 = (-0.1392 \quad 0.0192) \\ 90\ \%\ C.I &= -0.06 \pm 1.645 \cdot 0.04036 = (-0.1264 \quad 0.0064) \end{aligned}$$

   There is no clear evidence to say they are different.

3. *Prob 7.28*
   (a) $N = max\Big\{n \in Z^+ \ : \ \sqrt{\frac{p_1(1-p_1)}{n}} \le 0.01 \quad \sqrt{\frac{p_2(1-p_2)}{n}} \le 0.01\Big\} \Leftrightarrow n > 2401$
   (b) $N = max\Big\{n \in Z^+ \ : \ \sqrt{\frac{p_1(1-p_1)}{n}} \le p_1 \cdot 0.1 \quad \sqrt{\frac{p_2(1-p_2)}{n}} \le p_1 \cdot 0.1\Big\} \Leftrightarrow n > 3234$

4. *Prob 7.42*
   (a) $R = \frac{\bar{Y}}{\bar{X}} = \frac{\sum Y_i}{\sum X_i} = 31.25$
   (b) $S^2_R = \frac{1}{n}\Big(1 - \frac{n-1}{N-1}\Big)\frac{1}{\bar{X}^2}\Big(R^2 S^2_x + S^2_y - 2RS_{xy}\Big) \approx 0.01 \cdot \frac{1}{3.2^2} \cdot \Big(31.25^2 * 2.282828 + 1010.101 - 2 \cdot 31.25 \cdot 40.40404\Big) = 0.0.71954$ and $S_R = 0.8481$ The 95 % conficence interval is as usual.
   (c) $\widehat{\tau} = N \cdot \bar{Y} = 10^7$ and $s^2_{\widehat{\tau}} = 100000^2 \cdot s^2_{\bar{Y}} = 100000^2 \cdot 1010.101/100$. Therefor the 90 % C.I. is $10000000 \pm 1.645 \cdot 317820.9$.

5. *Prob 7.46*
   1. Population mean

$$\begin{aligned} \bar{X}_s &= w_1\bar{X}_1 + w_2\bar{X}_2 + w_3\bar{X}_3 = 177.24 \\ S^2_{\bar{x}_s} &= \sum_{l=1}^{2} \frac{1}{n_l}\Big(1 - \frac{n_l}{N_l}\Big)s^2_l \\ &= 219.5 \end{aligned}$$

   Therefore 90% C.I. is (152.8, 201.7),
   2. Population total

$$T_s = 2500 \cdot \bar{X}_s \qquad S_{T_s} = 2500 \cdot S_{\bar{x}_s} = 37040$$

   Thus the 90% C.I. is (381984, 504216).