Homework 1. Due in class, Wed Jan 13

1. We use the "Beta-Gamma calculus" so often, we'd like to make sure you know how it works. The Gamma density on $(0,\infty)$ is $\frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)}$ for $\alpha > 0$, a fixed parameter. The Beta density on $(0,1)$ is $\frac{x^{\gamma-1}(1-x)^{\delta-1}}{B(\gamma, \delta)}$ for $\gamma, \delta > 0$. Let $X, Y, Z$ be independent random variables having Gamma densities with respective parameters $\alpha, \beta, \gamma, \delta$. Show that $X/(X+Y), \frac{X+Y}{X+Y+Z}$ and $X+Y+Z$ are independent $\beta(\alpha, \gamma), \beta(\beta+\gamma, \delta)$, Gamma($\lambda+\gamma, \delta$) distributed. Can you give a conceptual reason for the independence?

2. An urn contains one red, one white, and one blue ball. Each time, a ball is chosen at random uniformly from the urn and replaced, together with a new ball of the same color. Let $X_1, X_2, X_3, \ldots$ denote the colors drawn on trials $1, 2, 3, \ldots$. Consider another process taking values in $\{R, W, B\}$. Pick $(p, q, r) \geq 0, p+q+r=1$ from the uniform distribution on the simplex. Fix this and draw repeatedly and independently from this distribution repeat the $R, W, or B$ as you draw $1, 2, or 3$. Let $Y_1, Y_2, Y_3, \ldots$ be the resulting process. Prove that, for any $A$

$$P(\bigcap_{i=1}^{\infty} Y_i = A) = P(\bigcap_{i=1}^{\infty} X_i = A)$$

3. Let $\lambda_1, \lambda_2, \lambda_3, \ldots$ be positive numbers with $\lambda = \sum \lambda_i < \infty$. Let $Z_i$ be independent random variables with parameters $\lambda_i$. Show that $S = \sum_{i=1}^{\infty} Z_i$ is finite almost surely. Define a random probability $Y = (Y_1, Y_2, \ldots)$ by $Y_i = Z_i/S$. The joint law of the $Y_i$ is called the Gamma distribution. Here is another construction:

Let $W_n \sim \text{Beta}(-\lambda, -\lambda)$, $W_2 \sim \text{Beta}(1, -\lambda)$, $W_3 \sim \text{Beta}(2, -\lambda)$. 

All independent form an infinite random probability by 'stick breaking':

$P_0 = W_1, P_1 = W_2(1-P_0), P_2 = W_3(1-P_0, \ldots.$

Show that $P = (P_0, P_1, \ldots)$ has a $\text{Dir}(\lambda_1, \lambda_2, \ldots)$ distribution.
NOTES ON BAYESIAN BOOKS.

There are now many reasonable Bayesian books; I put one of them which seems current & good on reserve for statistics 370. JEFF GILL (2008) BAYESIAN METHODS CHAPMAN AND HALL. This has a social science perspective but has good coverage of math and computing background.

A useful book which you can download from the web is E.T. JAYNES (1995) PROBABILITY THEORY: THE LOGIC OF SCIENCE; http://omega.albany.edu:8008/JaynesBook.html. This was my first introduction to BAYES; IT'S VERY DOWN TO EARTH AND WORTH WHILE. See also, DAVID MACKAY (2003) INFORMATION THEORY, INVERSE PROBLEMS AND LEARNING ALGORITHMS.

The 'standard' modern books are, BERGER, J. (1985) STATISTICAL DECISION THEORY AND BAYESIAN ANALYSIS 2nd ed. SPRINGER; and BERNARD, J. AND SMITH, A. (1994) BAYESIAN THEORY.

Two classics: JEFFREYS, H. (1961) THEORY OF PROBABILITY 3rd ed. OXFORD and LINDLEY, D.V. (1965) INTRODUCTION TO PROBABILITY AND STATISTICS FROM A BAYESIAN VIEWPOINT CAMBRIDGE. (the first course I ever taught was out of LINDLEY, its still good and JEFFREYS book is great)


A very good book on MONTE-CARLO METHODS FOR BAYESIAN ANALYSIS is CHEN, H. SHIOK, I. AND JIN, S. (2000) MONTE CARLO METHODS IN BAYESIAN COMPUTATION SPRINGER.

Our library has a Bayesian section. See in particular the Series 'STUDIES IN BAYESIAN APPLICATION' Series R. KASS Ed. Also, THE SERIES OF CONFERENCE PROCEEDINGS BAYESIAN STATISTICS I - VII Ed by JOSÉ BERNARDO ET AL, FOR MANY APPLICATIONS & CURRENT RESEARCH.

More coming!