Statistics 200 Homework 6 Solutions

Due January 19, 2005

9.18 Only (a) and (c) are true, all the others are false.

(b) If the p-value is 0.03, the test accepts at significance level 0.02.
(d) The p-value is the smallest significance level at which the test rejects the null hypotheses.
(e) The p-value is not equal to the likelihood ratio.
(f) The p-value is 0.075, it’s greater than 0.05.

9.26 The MLE $\hat{\theta} = 0.0357$, $p_1(\hat{\theta}) = .25(2 + \hat{\theta})$, $p_2(\hat{\theta}) = .25(1 - \hat{\theta})$, $p_3(\hat{\theta}) = .25(1 - \hat{\theta})$, $p_4(\hat{\theta}) = .25\hat{\theta}$.

\[ X^2 = \sum_{i=1}^{4} \frac{(x_i - np_i(\hat{\theta}))^2}{np_i(\hat{\theta})} = 2.0155 \]

The degree of freedom is $4 - 1 - 1 = 2$. The null distribution is approximately chi-square with 2 df. The p-value is 0.365. At level $\alpha = 0.05$, we don’t reject the null hypothesis, therefore the model is well fit.

9.27

\[ X^2 = \sum_{i=1}^{3} \frac{(x_i - np_i(\hat{\theta}))^2}{np_i(\hat{\theta})} = 0.0060 \]

Under the null hypothesis, it has approximately chi-square distribution with one df. The p-value is 0.9384. The model is well fit.

9.28 Under the null hypothesis that the rate is constant, the number of suicides in each month has the distribution of a multinomial random variable with probabilities $p_i = (\# \text{ of days in each month})/365$

\[ X^2 = \sum_{i=1}^{12} \frac{(x_i - np_i)^2}{np_i} = 47.3653 \]

Compared with chi-square distribution with df 11, the p-value is 1.852011e-06. Therefore at level $\alpha = 0.05$, we reject the null hypothesis that the rate is constant. The rate is somewhat higher in summer, with the exception of July, and lower in the winter.
9.30 We know \( X_2 = n - X_1 \), \( p_2 = 1 - p_1 \),
\[
X^2 = \frac{(X_1 - np_1)^2}{np_1} + \frac{(X_2 - np_2)^2}{np_2} \\
= \frac{(X_1 - np_1)^2}{np_1} + \frac{(n - X_1 - n(1 - p_1))^2}{np_2} \\
= \frac{(X_1 - np_1)^2}{np_1} + \frac{(np_1 - X_1)^2}{np_2} \\
= \frac{(X_1 - np_1)^2}{np_1(1 - p_1)}
\]
Under the null hypothesis, by CLT,
\[
\frac{X_1 - np_1}{\sqrt{np_1(1 - p_1)}} \sim N(0, 1)
\]
Therefore,
\[
\frac{(X_1 - np_1)^2}{np_1(1 - p_1)} \sim \chi^2_1
\]

9.34
\[
-2 \log \Lambda = -2n \sum_{i=1}^{3} p_i(\hat{\theta}) \log \left( \frac{p_i(\theta_0)}{p_i(\hat{\theta})} \right) = 115.4972
\]
where \( \theta_0 = 0.5, \hat{\theta} = 0.77 \), the mle.
The null distribution is approximately \( \chi^2_1 \), the p-value is 0. So the null hypothesis is overwhelmingly rejected.

9.36
\[
Var(\hat{p}) = \frac{p(1 - p)}{n}, \quad \mu = E(\hat{p}) = p
\]
Let \( f(x) = \sin^{-1} \sqrt{x} \)
\[
f'(\mu) = \frac{1}{\sqrt{1 - \mu}} \cdot \frac{1}{2\sqrt{\mu}} = \frac{1}{\sqrt{1 - p}} \cdot \frac{1}{2\sqrt{p}} \\
f'(\mu)^2 = \frac{1}{4p(1 - p)}
\]
By Delta-Method,
\[
Var(Y) = Var(\hat{p})f'(\mu)^2 = \frac{p(1 - p)}{n} \cdot \frac{1}{4p(1 - p)} = \frac{1}{4n}
\]
\( Var(Y) \) is a constant independent of \( p \), therefore \( Y \) is variance-stabilizing.