

# Myths Companion

STAT 222  
week 2  
D Rogosin

M1 individual growth  $\xi_p(t) = f(\xi, t)$   
proportional growth to asymptote  $\lambda_p$

Amount of Change  $\xi_p(t) = \lambda_p - (\lambda_p - \xi_p(0)) e^{-\lambda_p t}$   
 $A_p(t, t+c) = [\lambda_p - \xi_p(0)] [1 - e^{-\lambda_p t}] e^{-\lambda_p c}$  trading places

M2  $X_{ip} = \xi_{ip} + \varepsilon$  (fallible score, see back)  
 $D = X_2 - X_1, E(D) = \xi_2 - \xi_1$

$\sigma_D^2 = (t_2 - t_1)^2 \sigma_{\xi}^2$   
individual differences in change

reliability  $\rho(D) = \frac{\sigma_D^2}{\sigma_D^2 + \sigma_{\varepsilon_2}^2 - \varepsilon_1}$

M3  $\rho_{\xi(t), \xi(t+c)}$  on back, depends on choice of  $t, t+c$

for  $\rho_{\xi, D} = 0$   $\rho_{\xi_1, \xi_2} = (1 + \sigma_D^2 / \sigma_{\xi_1}^2)^{-1/2}$

M4  $\rho_{\xi(t_1), \varepsilon}$  on back depends on  $t_1, -t_0$

Bias of  $r_{X, D}$   $E(r_{X, D}) = \rho_{\xi(t_1), \varepsilon} \sqrt{\rho(X_1) \rho(D)} = \frac{\sigma_{\varepsilon_1}^2 - \sigma_{\varepsilon_1, \varepsilon_2}}{\sigma_X \sigma_D}$   
proportional bias additional downward

M5 Standardized Tautology

$$\frac{E(\xi_2 | \xi_1 = c) - \mu_{\xi_2}}{\sigma_{\xi_2}} < \frac{c - \mu_{\xi_1}}{\sigma_{\xi_1}} \Rightarrow \rho_{\xi_1, \xi_2} < 1$$

In metric of data  $E(\xi_2 | \xi_1 = c) - \mu_{\xi_2} < c - \mu_{\xi_1}$   
 $\Rightarrow \rho_{\xi_1, D} < 0$

M6/7 resid change in sample  $X_2 - X_1$

bias poor reliability

correlation  $\rho[A \cdot \xi(t_1)] \omega$  vs  $\rho \omega$   
 $A \cdot \xi(t_1) = \xi(t_2) \cdot \xi(t_1)$

# Properties (Moments of Observables) of Collections of Growth Curves

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for indiv  $p$   $\xi_p(t) = \xi_p(0) + \theta_p t$   $t_i (i=1, \dots, T)$   
 $p (p=1, \dots, n)$

centering, scale  
 $t^0 = -\sigma_{\xi(0)}/\sigma_{\theta}$   
 $P_{\xi(t^0)} = 0, \text{ min var}(\xi)$   
Scale  $K = \sigma_{\xi(t^0)}/\sigma_{\theta}$   
time metric

"centering"

$$\xi_p(t) = \xi_p(t^0) + \theta_p(t - t^0)$$

## Moments

Covariance  $\sigma_{\xi(t_1)\xi(t_2)} = (t_1 - t^0)(t_2 - t^0)\sigma_{\theta}^2 + \sigma_{\xi(t^0)}^2$

Variance  $\sigma_{\xi(t)}^2 = \sigma_{\xi(t^0)}^2 + ((t - t^0)/K)^2 \sigma_{\theta}^2$   
 $\sigma_{\xi(t)}^2 / \sigma_{\xi(t^0)}^2 = 1 + \left(\frac{t - t^0}{K}\right)^2$

Correl change, initial status

$$\rho_{\xi(t)} = \frac{t - t^0}{[K^2 + (t - t^0)^2]^{1/2}}$$

exogenous var  $w$

$$\rho_{w\xi(t)} = \frac{(t - t^0)\rho_{w\theta} + K\rho_{w\xi(t^0)}}{[K^2 + (t - t^0)^2]^{1/2}}$$

where  $t^u = t^0 + K\left(\frac{\rho_{w\theta}}{\rho_{w\xi(t^0)}}\right)$   $t^l = t^0 - K\left(\frac{\rho_{w\xi(t^0)}}{\rho_{w\theta}}\right)$

Week 1 example: (p.64)  $\theta \sim U[1, 9]$ ,  $\xi(t^0) \sim U[38, 62]$   
 $t^0 = 2$   $\sigma_{\theta}^2 = 5.333$   $\sigma_{\xi(t^0)}^2 = 48$   $\rho_{w\theta} = 0$   $\rho_{w\xi(t^0)} =$

at time  $t_i$   $X_{ip} = \xi_{ip} + \epsilon$   $\epsilon \sim (0, \sigma_{\epsilon}^2)$  errors in variables  
week 1 ex  $\sigma_{\epsilon}^2 = 10$