Myths Companion

STAT 222
week 2
D Rogosa

M1 individual growth $\xi_p(t) = f(\xi, t)$

proportional growth to asymptote λ_p

Amount of Change $\quad \xi_p(t) = \lambda_p - (\lambda_p - \xi_p(0))e^{-\gamma_p t}$

$A_p(t, t+c) = [\lambda_p - \xi_p(0)][1 - e^{-\gamma_p t}]e^{-\gamma_p(t)}$ $\quad$ trading places

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M2 $X_{ip} = \xi_{ip} + \epsilon$ (fallible score, see back)

$D = X_2 - X_1$, $E(D) = \xi_2 - \xi_1$

$\sigma_\Delta^2 = (t_2 - t_1)^2 \sigma_\theta^2$ individual differences in change $\quad$ reliability $\rho(D) = \dfrac{\sigma_\Delta^2}{\sigma_\Delta^2 + \sigma^2_{\epsilon_2 - \epsilon_1}}$

M3 $\rho_{\xi(t)\xi(t+c)}$ on back, depends on choice of t, $t+c$

for $\rho_{\xi_1 \Delta} = 0 \quad \rho_{\xi_1 \xi_2} = (1 + \sigma_\Delta^2/\sigma_{\xi_1}^2)^{-1/2}$

M4 $\rho_{\xi(t_1)\theta}$ on back $\quad$ depends on $t_1 - t^\circ$

Bias of $r_{X_1 D}$ $\quad E(r_{X_1 D}) = \underbrace{\rho_{\xi(t_1)\theta}\sqrt{\rho(X_1)\rho(D)}}_{\text{proportional bias}} - \underbrace{\dfrac{\sigma_{\epsilon_1}^2 - \sigma_{\epsilon_1 \epsilon_2}}{\sigma_{X_1}\sigma_D}}_{\text{additional downward}}$

M5 Standardized Tautology

$$\frac{E(\xi_2 \mid \xi_1 = C) - \mu_{\xi_2}}{\sigma_{\xi_2}} < \frac{C - \mu_{\xi_1}}{\sigma_{\xi_1}} \Rightarrow \rho_{\xi_1 \xi_2} < 1$$

In metric of data $E(\xi_2 \mid \xi_1 = C) - \mu_{\xi_2} < C - \mu_{\xi_1}$

$\Rightarrow \rho_{\xi_1 \Delta} < 0$

M6/7 resid change in sample $X_2 \cdot X_1$ $\quad$ bias poor reliability

correlation $\rho_{[\Delta \cdot \xi(t_1)]W}$ vs $\rho_{\theta W}$

$\Delta \cdot \xi(t_1) = \xi(t_2) \cdot \xi(t_1)$

Properties (Moments of Observables) of Collections of Growth Curves

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for indiv p $\quad \xi_p(t) = \xi_p(0) + \theta_p t \qquad t_i \ (i = 1, \dots T)$
$p \ (p = 1, \dots n)$

centering, scale

$t^o = -\sigma_{\xi(0)\theta} / \sigma^2_\theta$

$\rho_{\xi(t^o)\theta} = 0$, min $Var(\xi)$

Scale $K = \sigma_{\xi(t^o)} / \sigma_\theta$

time metric

"centering"

$$\xi_p(t) = \xi_p(t^o) + \theta_p(t - t^o)$$

Moments

Covariance $\sigma_{\xi(t_1)\xi(t_2)} =$

$$(t_1 - t^o)(t_2 - t^o)\sigma^2_\theta + \sigma^2_{\xi(t^o)}$$

Variance $\quad \sigma^2_{\xi(t)} = \sigma^2_{\xi(t^o)} + ((t - t^o)/K)^2 \sigma^2_{\xi(t^o)}$

$$\sigma^2_{\xi(t)} / \sigma^2_{\xi(t^o)} = 1 + \left(\frac{t - t^o}{K}\right)^2$$

Correl change, initial status $\quad \rho_{\theta\xi(t)} = \dfrac{t - t^o}{[K^2 + (t - t^o)^2]^{1/2}}$

exogenous var w

$$\rho_{w\xi(t)} = \frac{(t - t^o)\rho_{w\theta} + K\rho_{w\xi(t^o)}}{[K^2 + (t - t^o)^2]^{1/2}}$$

where $\quad t^u = t^o + K\left(\dfrac{\rho_{w\theta}}{\rho_{w\xi(t^o)}}\right) \qquad t^l = t^o - K\left(\dfrac{\rho_{w\xi(t^o)}}{\rho_{w\theta}}\right)$

Week 1 example: (p.64) $\theta \sim U[1,9]$, $\xi(t^o) \sim U[38,62]$

$t^o = 2 \quad \sigma^2_\theta = 5.333 \quad \sigma^2_{\xi(t^o)} = 48 \qquad \rho_{w\theta} = 0 \quad \rho_{w\xi(t^o)} =$

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at time t_i $\quad X_{ip} = \xi_{ip} + \epsilon \qquad \epsilon \sim (0, \sigma^2_\epsilon)$ $\qquad$ errors in variables

week 1 ex $\sigma^2_\epsilon = 10$