Models for Lat Vent trajectories 4/14/12
see linked plots, Males Females

Attributes for each individual

Gender: sexM = 1 if male

Age Mn: average age of observation (i.e. if obs at age 40 42 46 48
Age Mn = 44
Age C: observation times for an individual centered for Age Mn
(i.e. for above, Age C = {4, -2, 2, 44
for that individual)

Level I model (for individual i)
observations j

Vol_{ij} = \alpha_{0i} + \alpha_{1i} \text{age C}_{ij} + \varepsilon_{ij}

\alpha_{0i} mean Vol level for i
\alpha_{1i} slope of Vol trajectory on age for i

Level II model (basic)

level
\alpha_0 = \gamma_{00} + \gamma_{01} \text{age Mn} + \gamma_{02} \text{sexM} + u_0

slope \alpha_1 = \gamma_{10} + \gamma_{11} \text{age Mn} + \gamma_{12} \text{sexM} + u_1

both mean Vol and slope increase w/ Age Mn and constant displacement for gender (R > 0)
Combined Model (estimation model)

substitute $x_0, x_1$ level II into level I

$Vol \sim \gamma_0$ ageMn + $\gamma_2$ sexM + $\gamma_{10}$ ageC

+ $\gamma_{11}$ ageMn $\times$ ageC + $\gamma_{12}$ sexM $\times$ ageC

+ [$3 + \mu_0 + \mu_1$ terms]

(* indicates multiplication)

* notation, $A*B$ indicates $A, B, A:B$

collect terms for lmer model

$Vol \sim$ ageMn $*$ ageC $+$ sexM $*$ ageC

summary for lmer object

<table>
<thead>
<tr>
<th>param</th>
<th>lmer</th>
<th>interp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{00}$</td>
<td>model intercept</td>
<td>Increase in $Vol$ by unit</td>
</tr>
<tr>
<td>$\delta_{01}$</td>
<td>ageMn</td>
<td>increase ageMn</td>
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<tr>
<td>$\delta_{02}$</td>
<td>sexM</td>
<td>increment in $Vol$ for males</td>
</tr>
<tr>
<td>$\gamma_{10}$</td>
<td>ageC</td>
<td>value of $\alpha_1$ for $\text{age}=0$, Fem</td>
</tr>
<tr>
<td>$\gamma_{11}$</td>
<td>ageMn $\times$ ageC</td>
<td>increase in $\alpha_i$ for unit increase ageMn</td>
</tr>
<tr>
<td>$\gamma_{12}$</td>
<td>sexM $\times$ ageC</td>
<td>increase in $\alpha_i$ (slope) for Male</td>
</tr>
</tbody>
</table>
Even more, expand model to allow increase in $\alpha_{1,c}$ (slope) for Male to depend on age ($\text{ageMn}$).

\[ \gamma_{13}: \text{gender difference in } \alpha_1 \text{ on ageMn gradient} \]

**Level II model extension**

\[ \alpha_1 = \delta_{10} + \delta_{11} \text{ageMn} + \delta_{12} \text{sexM} + \gamma_{13} \text{sexM} \times \text{ageMn} \]

**Combined (estimation) model**

\[ \text{Vol} \sim \text{sexM} \times \text{ageMn} \times \text{ageC} - \text{sexM} : \text{ageMn} \]

or equiv

\[ \text{Vol} \sim \text{ageMn} \times \text{ageC} + \text{sexM} \times \text{ageC} + \text{sexM} : \text{ageMn} : \text{ageC} \]