

## **Section 3–Inference using Bootstrap Resampling**

### **Bootstrap Confidence Intervals**

Bootstrap estimation is provided for the following parameters

$\mu_{\theta}$  Mean(Rate)

Median( $\theta$ )

$\sigma_{\theta}^2$  var(Rate)

$\rho(\hat{\theta})$  rel(Rate)

$\rho_{\eta(t)\theta}$  Corr(Rate, Initial status)

$\beta_{\theta W}$  Beta(Rate, W)

$\beta_{\eta(t)W}$  Beta(Eta(T1), W)

For each parameter, there are two sections of the display:

1. **Estimate (Est)** contains the parameter estimate from the mixed-model analysis—either from fixed or random effects output—given the generic “mle” label; the mean over the bootstrap resamples of that estimate (mean.boot); and the standard deviation of that estimate over the bootstrap resamples (se.boot).

In addition some parameters include a se.SAS entry, which gives (for comparison) the standard error provided by the PROC MIXED run on the data. Also, regression coefficients include a se.OLS entry, which gives (for comparison) the standard error provided by the OLS regressions shown in the descriptive portions of the Timepath97 output.

### **2. Confidence Interval (CI plus BCa Detail)**

The CI portion of the section gives the Standard Interval—symmetric interval using  $N(0,1)$  and se.boot Percentile—based on the unadjusted percentiles of the bootstrap distribution.

BCa—Bias Corrected and Accelerated intervals: BCa Detail gives the coefficients and percentile shifting used in the procedure.

The columns in CI give the endpoints for various percentiles—i.e. a 90% interval would be constructed with endpoints obtained from the .05 and .950 columns. Also Shape and Length for the 95% interval is given.

\* Number of Bootstraps: 4000

\* Mean(Rate) \*

Est:	mle	mean.boot	se.boot			
	36.4481	36.4451	0.44428			

CI:	0.025	0.050	0.950	0.975	Shape95	Length95
Standard	35.5773	35.7173	37.1788	37.3188	1	1.74
Percentile	35.566	35.709	37.1855	37.3192	0.99	1.75
BCa	35.576	35.7168	37.1973	37.3297	1.01	1.75
BCa Detail:	a	z0	0.025	0.050	0.950	0.975
	-214E-7	0.01598	0.02692	0.05338	0.9532	0.97681

\* Median(Rate) \*

Est:	mle	mean.boot	se.boot			
	36.4167	36.4546	0.31607			

CI:	0.025	0.050	0.950	0.975	Shape95	Length95
Standard	35.7972	35.8968	36.9365	37.0361	1	1.24
Percentile	35.8333	35.8571	36.9524	37.0238	1.04	1.19
BCa	35.8333	35.8571	36.9762	37.0238	1.04	1.19
BCa Detail:	a	z0	0.025	0.050	0.950	0.975
	-349E-7	0.01379	0.02665	0.0529	0.95277	0.97656

\* var(Rate) \*

Est:	mle	mean.boot	se.boot	se.SAS		
	46.2293	45.8976	5.88593	.		

CI:	0.025	0.050	0.950	0.975	Shape95	Length95
Standard	34.6931	36.5478	55.9108	57.7655	1	23.1
Satterthwaite	38.1469	39.3323	55.2507	57.198		
Percentile	34.9386	36.6664	56.1952	58.4156	1.08	23.5
BCa	36.7909	38.3039	58.5592	61.2847	1.6	24.5
BCa Detail:	a	z0	0.025	0.050	0.950	0.975
	0.03941	0.10799	0.05283	0.08995	0.97676	0.99085

\* Rel(Rate) \*

Est:	mle	mean.boot	se.boot			
	0.82795	0.82497	0.01883			

CI:	0.025	0.050	0.950	0.975	Shape95	Length95
Standard	0.79103	0.79697	0.85893	0.86486	1	0.07
Percentile	0.78492	0.79225	0.85433	0.85853	0.71	0.07
BCa	0.79197	0.79795	0.85825	0.8629	0.97	0.07
BCa Detail:	a	z0	0.025	0.050	0.950	0.975
	0.03499	0.09162	0.04825	0.08355	0.97384	0.98933

\* Corr(Rate, Initial status) \*

Est:	mle	mean.boot	se.boot			
	0.651	0.66128	0.09068			

CI:	0.025	0.050	0.950	0.975	Shape95	Length95
Standard	0.47328	0.50185	0.80015	0.82872	1	0.36
Percentile	0.48425	0.51163	0.80785	0.83411	1.1	0.35
BCa	0.45642	0.48381	0.78466	0.80622	0.8	0.35
BCa Detail:	a	z0	0.025	0.050	0.950	0.975
	-0.0354	-0.1055	0.0098	0.02434	0.91218	0.94895

\* Beta(Rate, W) \*

Est:	mle	mean.boot	se.boot	se.SAS	se.OLS	
	0.33569	0.33534	0.02736	.	0.02537	

CI:	0.025	0.050	0.950	0.975	Shape95	Length95
Standard	0.28207	0.29069	0.3807	0.38932	1	0.11
Percentile	0.28127	0.28959	0.37939	0.38898	0.98	0.11
BCa	0.28184	0.29044	0.38059	0.39029	1.01	0.11
BCa Detail:	a	z0	0.025	0.050	0.950	0.975
	0.00921	0.00501	0.02772	0.05365	0.95356	0.97758

\* Beta(Eta(TI), W) \*

Est:	mle	mean.boot	se.boot	se.SAS	se.OLS	
	0.82948	0.83143	0.07881	.	0.08258	

CI:	0.025	0.050	0.950	0.975	Shape95	Length95
Standard	0.67502	0.69985	0.9591	0.98394	1	0.31
Percentile	0.67393	0.70195	0.9641	0.98557	1	0.31
BCa	0.66749	0.69527	0.95871	0.98081	0.93	0.31
BCa Detail:	a	z0	0.025	0.050	0.950	0.975
	-0.0129	-0.0213	0.01996	0.04235	0.94167	0.96926