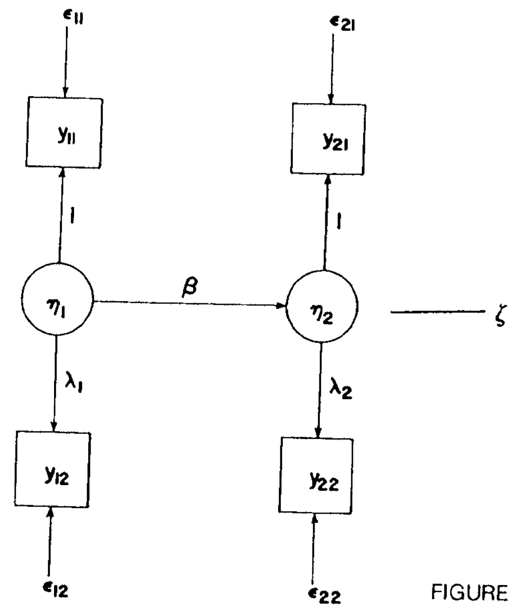

Structural Equation Models

Joreskog

FIGURE 11.4. **A two-wave–two-variable model.**

equation

$$\eta_2 = \beta\eta_1 + \zeta, \tag{28}$$

the regression of η_2 on η_1. In particular, we are interested in whether $\beta = 1$ and ζ is small; that is, whether the same latent variables are measured on both occasions.

The measurement-model part of the model may be written as

indicators load on latent vars

$$\begin{pmatrix} y_{11} \\ y_{12} \\ y_{21} \\ y_{22} \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ \lambda_1 & 0 \\ 0 & 1 \\ 0 & \lambda_2 \end{bmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \begin{pmatrix} \epsilon_{11} \\ \epsilon_{12} \\ \epsilon_{21} \\ \epsilon_{22} \end{pmatrix}, \tag{29}$$

where it is assumed that η_1 and η_2 are measured in the same metric as y_{11} and y_{21}, respectively. This model is a special case of the general LISREL model with no x. In terms of LISREL, Eq. (28) may be interpreted, in accordance with Eq. (4), as

$$\begin{pmatrix} 1 & 0 \\ -\beta & 1 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix},$$

regression for latent vars

where $\zeta_1 = \eta_1$ and $\zeta_2 = \zeta$. Let $\mathbf{\Phi}$ be the covariance matrix of (η_1, η_2) and let $\mathbf{\Theta}$ be the covariance matrix of $(\epsilon_{11}, \epsilon_{12}, \epsilon_{21}, \epsilon_{22})$. If all the ϵ's are

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uncorrelated so that $\mathbf{\Theta}$ is diagonal, the covariance matrix of $(y_{11}, y$ $y_{22})$ is

Cov(y)

$$\Sigma = \begin{pmatrix} \phi_{11} + \theta_{11} & & & \\ \lambda_1\phi_{11} & \lambda_1{}^2\phi_{11} + \theta_{22} & & \\ \phi_{21} & \lambda_1\phi_{21} & \phi_{22} + \theta_{33} & \\ \lambda_2\phi_{21} & \lambda_1\lambda_2\phi_{21} & \lambda_2\phi_{22} & \lambda_2{}^2\phi_{22} + \theta_{44} \end{pmatrix},$$

Σ has 10 variances and covariances that are functions of nine para The model has one degree of freedom.

Often when the same variables are used repeatedly, there is a te for the corresponding errors (the ϵ's) to correlate over time (see pre sections on autoregressive models) because of memory and other effects. Hence, there is a need to generalize the preceding model to for correlations between ϵ_{11} and ϵ_{21} and also between ϵ_{12} and ϵ_{22} means that there will be two nonzero covariances θ_{31} and θ_{42} in $\mathbf{\Theta}$ model is shown in Figure 11.5. The covariance matrix of the ob variables will now be

Cov(y)

$$\Sigma = \begin{bmatrix} \phi_{11} + \theta_{11} & & & \\ \lambda_1\phi_{11} & \lambda_1{}^2\phi_{11} + \theta_{22} & & \\ \phi_{21} + \theta_{31} & \lambda_1\phi_{21} & \phi_{22} + \theta_{33} & \\ \lambda_2\phi_{21} & \lambda_1\lambda_2\phi_{21} + \theta_{42} & \lambda_2\phi_{22} & \lambda_2{}^2\phi_{22} + \theta_{44} \end{bmatrix}.$$

corr error

This Σ has its 10 independent elements expressed in terms of 11 param Hence, it is clear that the model is not identified. In fact, none of parameters are identified without further restrictions. The loading λ λ_2 may be multiplied by a constant and the ϕ's divided by the same con This does not change σ_{21}, σ_{32}, σ_{41}, and σ_{43}. The change in the oth may be compensated by adjusting the θ's additively. Hence, to mak model identified one must fix one λ or one ϕ at a nonzero value or at some arbitrary value. However, the *correlation* between η_1 and identified without any restrictions, since

$$\mathrm{Corr}(\eta_1, \eta_2) = [\phi_{21}^2/\phi_{11}\phi_{22}]^{1/2}$$
$$= [(\sigma_{32}\sigma_{41})/(\sigma_{21}\sigma_{43})]^{1/2}$$

This model may therefore be used to estimate this correlation coeffi and to test whether this is 1. The maximum-likelihood estimate o correlation coefficient is $[(s_{32}s_{41})/(s_{21}s_{43})]^{1/2}$. To make further use o model it is necessary to make some assumption about the nature o variables. For example, if it can be assumed that the two variables at occasion are τ-equivalent (see, e.g., Lord & Novick, 1968) we can set λ_1 and λ_2 equal to 1. Then the model can be estimated and tested with degree of freedom.

Sem p.2 Simplex 4/19

Stablity of Alienation

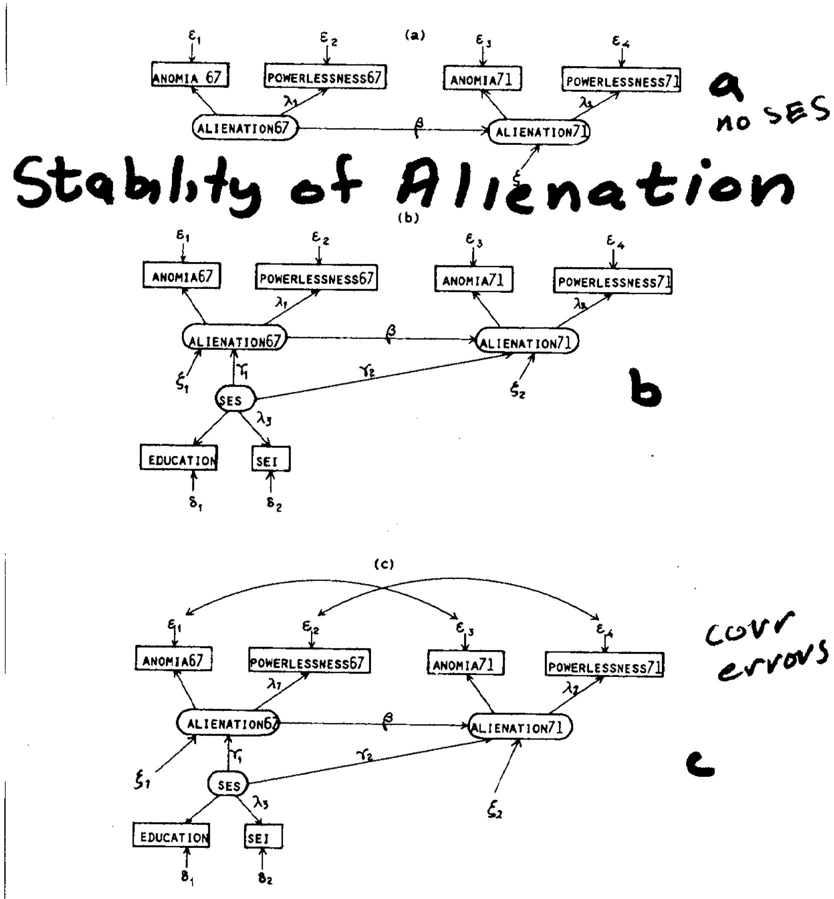

FIGURE 6. Models for study of stability of alienation.

TABLE 1

Parameter estimates for the models in Figure 6a–c (standard errors in parentheses).

Parameter	Model 6a	Model 6b	Model 6c
λ_1	0.815 (0.040)	0.888 (0.041)	0.979 (0.062)
λ_2	0.847 (0.042)	0.849 (0.040)	0.922 (0.059)
λ_3	–	5.331 (0.430)	5.221 (0.422)
β	0.789 (0.044)	0.705 (0.054)	0.607 (0.051)
γ_1	–	−0.614 (0.056)	−0.575 (0.056)
γ_2	–	−0.174 (0.054)	−0.227 (0.052)
ψ_{11}	–	5.307 (0.473)	4.847 (0.468)
ψ_{22}	4.085 (0.432)	3.742 (0.388)	4.089 (0.405)
ϕ	–	6.663 (0.641)	6.803 (0.650)
σ_{δ_1}	–	1.717 (0.145)	1.675 (0.151)
σ_{δ_2}	–	16.153 (0.565)	16.273 (0.558)
σ_{ϵ_1}	1.906 (0.097)	2.004 (0.086)	2.176 (0.104)
σ_{ϵ_2}	1.865 (0.077)	1.786 (0.076)	1.602 (0.126)
σ_{ϵ_3}	1.827 (0.109)	1.923 (0.097)	2.098 (0.123)
σ_{ϵ_4}	1.969 (0.077)	1.904 (0.077)	1.754 (0.124)
corr (ϵ_1, ϵ_3)	–	–	0.356 (0.047)
corr (ϵ_2, ϵ_4)	–	–	0.121 (0.082)
χ^2	61.155	71.544	4.770
d.f.	1	6	4

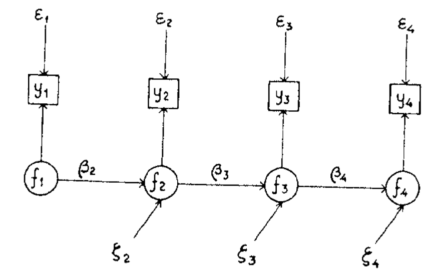

FIGURE 7. A simplex model.

The simplex model can be put into the LISREL format, with no x. We write

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} + \begin{pmatrix} 0 \\ \varepsilon_2 \\ \varepsilon_3 \\ 0 \end{pmatrix}, \tag{41}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -\beta_2 & 1 & 0 & 0 \\ 0 & -\beta_3 & 1 & 0 \\ 0 & 0 & -\beta_4 & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \end{bmatrix}. \tag{42}$$

covariance matrix of y_1, y_2, y_3 and y_4 is

$$\Sigma = \begin{bmatrix} \phi_1 + \theta_1^2 & & & \\ \beta_2\phi_1 & \phi_2 + \theta_2^2 & & \\ \beta_2\beta_3\phi_1 & \beta_3\phi_2 & \phi_3 + \theta_3^2 & \\ \beta_2\beta_3\beta_4\phi_1 & \beta_3\beta_4\phi_2 & \beta_4\phi_3 & \phi_4 + \theta_4^2 \end{bmatrix}. \tag{40}$$

cf Satisfying a simplex structure is simpler than it should be