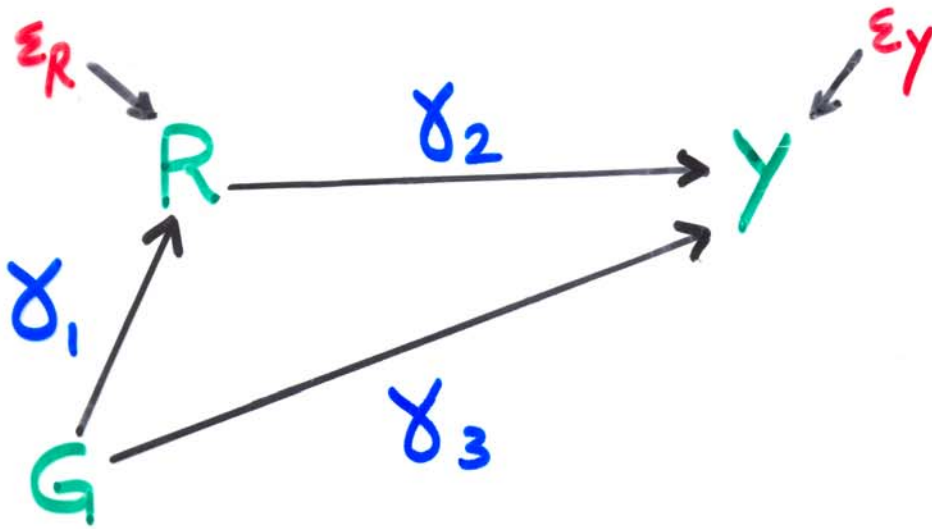


Path Analysis for Y, R, G



Path Regressions

R on G

Y on R, G

Path Analysis Results under ALICE

$$\delta_1 = \rho$$

$G \rightarrow R$

$$\delta_2 = \beta + \delta$$

$R \rightarrow Y$

$$\delta_3 = \gamma - \rho\delta$$

$G \rightarrow Y$

for $\delta = \text{cov}(Y_{c0}(u), R_c(u)) / \text{VAR}(R_c(u))$

Causal Effects	ALICE	Path Analysis
Overall	$\gamma + \beta\rho$	$\delta_3 + \delta_1\delta_2 =$ $\gamma + \beta\rho$
<u>Indirect</u> Direct	$\beta\rho/\gamma$	$\delta_1\delta_2/\delta_3 =$ $\rho(\beta + \delta)/(\gamma - \rho\delta)$

Examples under ALICE

$$\beta = 3, \gamma = 1, \text{Corr}(R_c, Y_{co}) = .75$$

$$\text{Var}(Y_{co} = 64) \quad \text{Var}(R_c = 4)$$

	$\rho = 1$ (.5 sel effect)	$\rho = 3$ (1+ sel effect)
$G \rightarrow R$ γ_1	1	3
$R \rightarrow Y$ γ_2	6	6
$G \rightarrow Y$ γ_3	-2	-8
ALICE ind/div	3	9
PATH ind/div	-3	-2.25

Even with random assignment to and control, path regressions do not recover the individual level model.

Example:

For individual level model:

$$\beta(u) \sim U[2, 8] \quad \gamma = 2 \quad \rho = 8$$

$$\mu_\beta = 5$$

Path analysis regressions give ($n=10,000$)

$$\hat{\delta}_2 = 8.57 \quad \hat{\delta}_3 = -26.3 \quad \hat{\delta}_1 = 8.01$$

Ratio of "indirect" to "direct" effects:

$$\frac{\mu_{\beta P}}{\gamma} = 20$$

$$\frac{\hat{\delta}_1 \hat{\delta}_2}{\hat{\delta}_3} = -2.6$$

Artificial Data (Quadratic Response)

	Y	R	G	Variance
Y	1			2934
R	.86	1		17.8
G	.78	.94	1	

Path Analysis gives:

$$\hat{\delta}_2 = 14.37$$

$$\hat{\delta}_3 = -29.4$$

$$\hat{\delta}_1 = 7.95$$

Data generation parameters

$$\mu_{\beta_1} + \mu_{\beta_2} \bar{R} = 6.8$$

$$\beta_1 \sim U[2, 4]; \beta_2 \sim U[.1, .3]$$

$$\gamma = 2$$

$$\rho = 8$$

Quadratic Response

$$Y(u) = Y_0(u) + \beta_1(u)R(u) + \beta_2(u)[R(u)]^2$$

More typical application in nonexperimental setting.

Level of encouragement (G)

determined by home environment, i.e. outside experimental control.

Failure of path analysis in a simple experiment should strike fear into such applications.