Section 1– Descriptive analyses of growth rates.

Individual OLS Fits

The most basic step in the analysis is the fitting of a straight-line growth curve (the regression of $Y$ on $t$ for each $p$) by ordinary least-squares.

Individual OLS Fits displays for each individual (rows) the (columns):

<table>
<thead>
<tr>
<th>ID</th>
<th>RATE (empirical rate; OLS estimate of $\theta_p$, slope $Y$ on $t$ fit)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>INIT_LVL (level for $Y$ on $t$ fit evaluated at the first anchor time point)</td>
</tr>
<tr>
<td></td>
<td>MSR (residual variance for $Y$ on $t$ fit)</td>
</tr>
<tr>
<td></td>
<td>RSQ (squared multiple correlation for $Y$ on $t$ fit)</td>
</tr>
<tr>
<td></td>
<td>$w$ (value of exogenous variable)</td>
</tr>
</tbody>
</table>

Cross-sectional Description:

For synchronous (same times of observation for each individual) data sets (in which it makes sense to talk about time-1 etc observations) cross-sectional descriptive summaries are provided.

(In data sets such as smearmiss, this output is automatically not computed).

Cross-sectional means and between-wave correlations for the $Y(t_i)$ (adding $\hat{\theta}$ and $W$)

OLS Fits: Descriptive Statistics

Estimation of the straight-line growth model allows comparisons of rates of change across individuals. Stem-and-leaf diagrams, boxplots, and the five-number summaries of the empirical rates are useful ways to describe both typical rates of learning and the heterogeneity across individuals. Displayed in this section are descriptive statistics for the quantities listed under Individual OLS Fits ($\text{RATE\ INIT_LVL\ MSR\ RSQ\ w}$) plus individual values for the Foulkes-Davis tracking index ($\text{Gamma}$). (Foulkes-Davis index of tracking estimated from a count of the number of intersections that each individual trajectory has with the other individuals; for each individual $\hat{\gamma}_p$ is one minus the number of intersections over $n - 1$. Individuals with a low value of $\hat{\gamma}_p$ are those whose relative standing changes considerably over the time period.)
Stem-and-leaf diagrams and accompanying boxplots are displayed for \( \text{RATE} \) (Empirical rate), \( \text{INIT_LVL} \) (Fitted Initial Level), and \( W \) (Exogenous Variable).

**OLS Fitted Values for Anchor Times**
Regardless of whether the Cross-sectional Description for (synchronous data) is printed above, descriptive (cross-sectional) statistics and between-wave correlations are provided for the following measures: \( TA_{FIT_i} \) values of the individual fits for each of the specified anchor time points, plus \( \text{RATE} \) \( W \).

**Rate and Fitted Initial Level Scatter plot** of \( \text{RATE} \) vs \( \text{INIT_LVL} \)
To provide some descriptive augmentation to the parameter estimate for the correlation \( \rho_{n(t)\theta} \) (correlation between change \( \theta_p \) and true initial status \( \eta_p(t_I) \), where \( t_I \) indicates a time of initial status designated by the first time anchor point).

**OLS Theta-hat on W Regression** When \( W \) is present, OLS regression and corresponding scatterplot is given—provides graphical diagnostic and shows correspondence (both point estimate and standard error) with fixed-effects estimates from mixed-model estimation (exact match for complete synchronous data).

**OLS Fitted Initial Level on W Regression** When \( W \) is present, OLS regression is given—provides graphical diagnostic and shows correspondence (both point estimate and standard error) with fixed-effects estimates from mixed-model estimation (exact match for complete synchronous data).
Section 2– Parameter Estimates

Parameter Estimates presents a collection of parameter estimates based on the growth curve model.

The first parameters listed are "typical" rates of change $\mu_0$ or median($\theta$), and a measure of heterogeneity $\sigma^2_\theta$, the variance of the $\theta_p$. The estimate of the reliability $\rho(\hat{\theta})$ is based on the estimate of $\sigma^2_\theta$ and estimate of Ave(errorvar( $\hat{\theta}$ )).

For each of the stated anchor times, the quantities estimated are:

- $\sigma^2_{\eta(t)}$
- $\rho_{\eta(t)\theta}$
- $\beta_{\theta \eta(t)}$

and Reliability $Y(t)$.

Using the chosen anchor time point, obtain desired estimate of the correlation between true rate of change and true initial status $\rho_{\eta(t)\theta}$. A good estimate of this correlation is made possible by the availability of multiple (e.g., 4 or more) longitudinal observations; a pervasive problem in the pre-test, post-test dominated measurement of change literature was that when only two observations were available, the only estimate was the correlation between observed change and observed initial status which may have large, usually negative, bias (see Rogosa et al. 1982).

The index of tracking $\gamma$ (Foulkes & Davis 1981) is used to assess the consistency of individual differences; the index is defined as the probability that two randomly chosen growth curves do not intersect. High values of $\gamma$ indicate high consistency of individual differences over time. The proportion of no intersections is accumulated over the $n$ individuals to produce an overall $\hat{\gamma}$ estimate. The standard error of $\hat{\gamma}$ can be obtained from a jackknife approximation given by Foulkes and Davis (1981) or by using bootstrap resampling.

Systematic individual differences in growth are indicated in these analyses by the quantity $\rho_{\theta W}$ or by $\beta_{W \theta}$; for example, non-zero values of $\beta_{W \theta}$ indicate that $W$ is a predictor of growth. Maximum likelihood estimates of these parameters are obtained from the fixed-effects output of PROC MIXED (see Tp Innards TAB).
Section 3–Inference using Bootstrap Resampling

Bootstrap Confidence Intervals
Bootstrap estimation is provided for the following parameters

\( \mu_0 \)  \( \text{Mean} \) (Rate)

Median(\( \theta \))

\( \sigma^2_0 \)  \( \text{var} \) (Rate)

\( \rho(\hat{\theta}) \)  \( \text{rel} \) (Rate)

\( \rho_{\eta(t_j)\theta} \)  \( \text{Corr} \) (Rate, Initial status)

\( \beta_{\theta W} \)  \( \text{Beta} \) (Rate, \( W \))

\( \beta_{\eta(t_j)W} \)  \( \text{Beta} \) (Etat(T1), \( W \))

For each parameter, there are two sections of the display:

1. **Estimate (Est)** contains the parameter estimate from the mixed-model analysis—either from fixed or random effects output—given the generic “mle” label; the mean over the bootstrap resamples of that estimate (mean.boot); and the standard deviation of that estimate over the bootstrap resamples (se.boot).

In addition some parameters include a se.SAS entry, which gives (for comparison) the standard error provided by the PROC MIXED run on the data. Also, regression coefficients include a se.OLS entry, which gives (for comparison) the standard error provided by the OLS regressions shown in the descriptive portions of the Timepath97 output.

2. **Confidence Interval (CI plus BCa Detail)**

The CI portion of the section gives the

Standard Interval—symmetric interval using \( N(0,1) \) and se.boot

Percentile—based on the unadjusted percentiles of the bootstrap distribution.

BCa–Bias Corrected and Accelerated intervals: BCa Detail gives the coefficients and percentile shifting used in the procedure.

The columns in CI give the endpoints for various percentiles—i.e. a 90% interval would be constructed with endpoints obtained from the .05 and .950 columns. Also Shape and Length for the 95% interval is given.