

Data Analysis and Parameter Estimation

Precursor: Descriptive Growth Curve Analyses

SFYS: fit Y on t regressions, describe resulting $\hat{\theta}_p$, fit $\hat{\theta}_p$ on W regr,
 Examples: WISC, frames 1-4; Ramus, frames 1-3; SmearMiss, frames 1-3.
 Even non-synchronous data, get variance comps and derived quants by
 approx method-of-moments (Rogosa-Saner 1995); works surprisingly well.

Maximum Likelihood estimation for parameters

Special, simple case; Complete, Synchronous Data.

ml estimation equations for full data in closed form (Blomqvist 1977)

example estimation of $\text{var}(\theta) \sigma_\theta^2$

MSR_p mean squared residual for OLS fit individual p ; $\hat{\sigma}^2$ is $\text{Ave}(MSR_p)$.

estimate for σ_θ^2 : $\hat{\sigma}_\theta^2 = \text{SS}(\hat{\theta}_p) / "n" - \hat{\sigma}^2 / \text{SSt}$,

reliability estimate for $\hat{\theta}_p$: $\hat{\rho}(\hat{\theta}) = \hat{\sigma}_\theta^2 / \text{SS}(\hat{\theta}_p) / "n"$

General strategy: get elements of 2x2 est. covariance matrix of θ
 and $\eta(0)$ for full or incomplete data. Common to **All programs** (LISREL
 HLM Tp) Tp: further substitute for derived quantities.

Also, fixed effects from separate run with W (when exists)–OLS equiv

properties of mle: bias, precision: Is reml best?

bias and mean-square-error : compare ML and REML

mle and reml simulation (50,000); complete synchronous data

		Estimation of σ_θ^2 var(theta) = 5.0	
		ML	REML
n			
10	4.37 [7.39]	4.99 [8.61]	
15	4.58 [5.06]	4.99 [5.59]	

MAJOR MESSAGES

1. OLS equivalences for fixed effects; Method-of-moments match for random effects
2. 2x2 covariance matrix ($\eta_p(0) \theta_p$)-- elements σ_θ^2 $\sigma_{\eta(0)}^2$ $\sigma_{\eta(0)\theta}$ --starting point for growth statistics
3. uncertainty, via s.e. and CI, reporting essential–for small (or medium) n, BCa intervals vs standard

From Growth Curves to Mixed(Random)-Effects Models

$$N_p(t) = N_p(0) + \sigma_p t$$

intercepts and slopes may differ across $p = 1, \dots, N$

Data $y_{pj} = N_p(t_{pj}) + \epsilon_{pj}$
 ordered times $j = 1 \dots T_p$ obs for p
 (missing or not)

No ω

$$E_p(\sigma_p) = \mu_\sigma$$

$$E_p(N_p(0)) = \mu_{N(0)}$$

Random

$$N_p(0) - \mu_{N(0)}$$

$$\sigma_p - \mu_\sigma$$

Fixed

$$\mu_{N(0)}$$

$$\mu_\sigma$$

Mixed Effects Model

$$Y = X\beta + Z\gamma + \epsilon$$

$N \sum_{p=1}^{T_p} \times 1$ $\Sigma T_p \times 2$ 2×1 $\Sigma T_p \times 2N$ $2N \times 1$ $2N \times 1$

fixed
ave. growth
curve

$$X = \begin{bmatrix} 1 & t_{p1} \\ \vdots & \vdots \\ 1 & t_{pT} \end{bmatrix}$$

$$\beta = \begin{bmatrix} \mu_{N(0)} \\ \mu_\sigma \end{bmatrix}$$

random

$$\gamma = \begin{bmatrix} N_p(0) - \mu_{N(0)} \\ \sigma_p - \mu_\sigma \end{bmatrix}$$

2 rows each p

blocks $Z = \begin{bmatrix} \begin{bmatrix} 1 & t_{p1} \\ \vdots & \vdots \\ 1 & t_{pT} \end{bmatrix} & \dots & \begin{bmatrix} \end{bmatrix} \end{bmatrix}$

$$\text{Var}(Y) = V = ZGZ' + R$$

block elements of G $\begin{bmatrix} \sigma_{N(0)}^2 & \sigma_{N(0)\sigma} \\ & \sigma_\sigma^2 \end{bmatrix}$

also D
aka

From Growth Curves to Mixed(Random)-Effects Models

with w

Fixed

$$E(\eta_i | w) = \mu_{\eta_i} + \beta_{\eta_i} w (w - \mu_w)$$

$$E(\sigma_i | w) = \mu_{\sigma} + \beta_{\sigma} w (w - \mu_w)$$

Random

$$\eta_p(w) - E(\eta_i | w)$$

$$\sigma_p - E(\sigma_i | w)$$

$$Y = X\beta + Z\gamma + \epsilon$$

expanding X
 $\sum T_p \times 4$

fixed

$$= \begin{bmatrix} 1 & t_{p1} & w_p & w_p * t_{p1} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & t_{pT_p} & w_p & w_p * t_{pT_p} \end{bmatrix}$$

$$\beta = \begin{bmatrix} \mu_{\eta_i} - \beta_{\eta_i} \mu_w \\ \mu_{\sigma} - \beta_{\sigma} \mu_w \\ \beta_{\eta_i} w \\ \beta_{\sigma} w \end{bmatrix}$$

int
time
 w
time $\times w$

random

$$\gamma = \begin{bmatrix} \eta_p(w) - E(\eta_i | w_p) \\ \sigma_p - E(\sigma_i | w_p) \\ \vdots \end{bmatrix}$$

$2N \times 1$

G-matrix contains conditional variances

$$\text{Var}(\sigma | w) \quad \text{Var}(\eta_i | w)$$

see NCFem, Frames 7, 8 etc

Timepath97



A dissemination project

Timepath97-parameter estimation

- * obtain estimates for growth curve quantities of interest from solutions (using Make, ODS facility for 6.11+) estimated covariance parameters give t^0 , κ , variances and derived quantities (see page 12) etc;
- * embed in jackboot.sas to obtain BCa confidence intervals for derived quantities (link to jackboot and docs on Timepath97 Web page)

Implementation of Estimation using SAS- PROC MIXED

(thanks to Neil Timm, Univ Pitt. & Russ Wolfinger, SAS Inc)

REML, ML etc available. (REML matches other E-M programs, e.g SmearMiss via HLM).

S-plus Alternative: lme-- Pinheiro & Bates, or further with nlme

<http://netlib.bell-labs.com/cm/ms/departments/sia/project/nlme/index.html>

put data in column form [ID, Y, t, W] Run PROC MIXED without and with W to obtain core quantities for parameter estimation

From no-W run obtain Covariance Parameter Matrix (G);

```
/* Proc mixed run */
proc mixed data=yt;
  class case;
  model y = time / s;
  random int time / type=un sub=case gcorr;
  make 'CovParms' out=untot;
  make 'SolutionF' out=solfout;
  %bystmt;
run;
```

fixed effects solution vector gives relations with W

```
proc mixed data=yt;
  class case;
  model y = time W time*W / s;
  random int time / type=un sub=case gcorr;
  make 'SolutionF' out=solfout;
  %bystmt;
run;
```

Raw SAS--- frames 7,8 NCFem; frame 7 Ramus; frames 7,8 Smearmiss;