Data Analysis and Parameter Estimation

Precursor: Descriptive Growth Curve Analyses

SFYS: fit Y on t regressions, describe resulting $\hat{\theta}_p$, fit $\hat{\theta}_p$ on W regr,
Examples: WISC, frames 1-4; Ramus, frames 1-3; SmearMiss, frames 1-3.
Even non-synchronous data, get variance comps and derived quants by approx method-of-moments (Rogosa-Saner 1995); works surprisingly well.

Maximum Likelihood estimation for parameters

Special, simple case; Complete, Synchronous Data.
ml estimation equations for full data in closed form (Blomqvist 1977)
example estimation of var(theta) σ^2_θ
MSR_p mean squared residual for OLS fit individual *p*; $\hat{\sigma}^2$ is Ave(MSR_p).
estimate for σ^2_θ : $\hat{\sigma}^2_\theta$ = SS($\hat{\theta}_p$)/"n" - $\hat{\sigma}^2/SSt$,
reliability estimate for $\hat{\theta}_p$: $\hat{\rho}(\hat{\theta})$ = $\hat{\sigma}^2_\theta$ /SS($\hat{\theta}_p$)/"n"

General strategy: get elements of 2x2 est. covariance matrix of θ and $\eta(0)$ for full or incomplete data. Common to **All programs** (**LISREL** HLM Tp) Tp: further substitute for derived quantities.
Also, fixed effects from separate run with W (when exists)–OLS equiv

properties of mle: bias, precision: Is reml best?
bias and mean-square-error : compare ML and REML
mle and reml simulation (50,000); complete synchronous data

Estimation of σ^2_θ var(theta) = 5.0

n	ML		REML	
10	4.37	[7.39]	4.99	[8.61]
15	4.58	[5.06]	4.99	[5.59]

MAJOR MESSAGES

1. OLS equivalences for fixed effects; Method-of-moments match for random effects

2. 2x2 covariance matrix ($\eta_p(0)$ θ_p)-- elements σ^2_θ $\sigma^2_{\eta(0)}$ $\sigma_{\eta(0)\theta}$ –starting point for growth statistics

3. uncertainty, via s.e. and CI, reporting essential–for small (or medium) n, BCa intervals vs standard

From Growth Curves to Mixed(Random)-Effects Models

$\eta_p(t) = \eta_p(0) + \theta_p t$

intercepts and slopes may differ across $p = 1, \ldots, N$

Data $Y_{pj} = \eta_p(t_{pj}) + \epsilon_{pj}$

ordered times $j = 1 \ldots T_p$ # obs for p (missing or not)

Now

$E_p(\theta_p) = \mu_\theta$

$E_p(\eta_p(0)) = \mu_{\eta(0)}$

Random

$\eta_p(0) - \mu_{\eta(0)}$

$\theta_p - \mu_\theta$

Fixed

$\mu_{\eta(0)}$

μ_θ

Mixed Effects Model

$$\underset{\sum_{p=1}^{N} T_p \times 1}{Y} = \underset{\sum T_p \times 2}{X} \underset{2 \times 1}{\beta} + \underset{\sum T_p \times 2N}{Z} \underset{2N \times 1}{\gamma} + \epsilon$$

fixed
ave. growth curve

$$X = \begin{bmatrix} \begin{bmatrix} 1 & t_{p1} \\ \vdots & \\ 1 & t_{pT} \end{bmatrix} \\ \\ \end{bmatrix} \qquad \beta = \begin{bmatrix} \mu_{\eta(0)} \\ \mu_\theta \end{bmatrix}$$

random

$$\underset{\text{2 rows each } p}{\gamma} = \begin{bmatrix} \begin{bmatrix} \eta_p(0) - \mu_{\eta(0)} \\ \theta_p - \mu_\theta \end{bmatrix} \\ \\ \end{bmatrix}$$

blocks

$$Z = \begin{bmatrix} \begin{bmatrix} 1 & t_{p1} \\ \vdots & \\ 1 & t_{pT} \end{bmatrix} & & \\ & \ddots & \\ & & [\quad] \end{bmatrix}$$

$$Var(Y) = V = Z \underset{\text{also D aka}}{G} Z' + R$$

block elements of G $\begin{bmatrix} \sigma^2_{\eta(0)} & \sigma_{\eta(0)\theta} \\ & \sigma^2_\theta \end{bmatrix}$

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With W

Fixed

$$E(\eta(0)|W) = \mu_{\eta(0)} + \beta_{\eta(0)W}(W - \mu_W)$$

$$E(\theta|W) = \mu_\theta + \beta_{\theta W}(W - \mu_W)$$

Random

$$\eta_p(0) - E(\eta(0)|W)$$

$$\theta_p - E(\theta|W)$$

$$Y = X\beta + Z\gamma + \varepsilon$$

fixed

expanding

$$\underset{\Sigma T_p \times 4}{X} = \begin{bmatrix} \begin{bmatrix} 1 & t_{pj} & w_p & w_p * t_{pj} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & t_{pT_p} & w_p & w_p * t_{pT_p} \end{bmatrix} \\ \\ \end{bmatrix}$$

$$\underset{4\times 1}{\beta} = \begin{array}{c} \begin{bmatrix} \mu_{\eta(0)} - \beta_{\eta(0)W}\,\mu_W \\ \mu_\theta - \beta_{\theta W}\,\mu_W \\ \beta_{\eta(0)W} \\ \beta_{\theta W} \end{bmatrix} \end{array} \begin{array}{l} \text{int} \\ \text{time} \\ W \\ \text{time}*W \end{array}$$

random

$$\underset{2N\times 1}{\gamma} = \begin{bmatrix} \begin{bmatrix} \eta_p(0) - E(\eta(0)|w_p) \\ \theta_p - E(\theta|w_p) \end{bmatrix} \\ \vdots \end{bmatrix}$$

G-matrix contains conditional variances

$Var(\theta|W)$ $\quad Var(\eta(0)|W)$

See NCFem, Frames 7, 8 etc

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Timepath97

A dissemination project

Timepath97–parameter estimation

*****obtain estimates for growth curve quantities of interest from solutions (using Make, ODS facility for 6.11+) estimated covariance parameters give t^o , κ, variances and derived quantities (see page 12) etc;

*****embed in jackboot.sas to obtain BCa confidence intervals for derived quantities (link to jackboot and docs on Timepath97 Web page)

Implementation of Estimation using SAS- PROC MIXED

(thanks to Neil Timm, Univ Pitt. & Russ Wolfinger, SAS Inc)
REML , ML etc available. (REML matches other E-M programs, e.g SmearMiss via HLM).

S-plus Alternative: `lme`-- Pinheiro & Bates, or further with `nlme`
http://netlib.bell-labs.com/cm/ms/departments/sia/project/nlme/index.html

put data in column form [ID, Y, t, W] Run PROC MIXED without and with W to obtain core quantities for parameter estimation
From no-W run obtain Covariance Parameter Matrix (G);

```
/* Proc mixed run */
 proc mixed data=yt;
   class case;
   model y = time / s;
   random int time / type=un sub=case gcorr;
   make 'CovParms' out=untot;
   make 'SolutionF' out=solfout;
   %bystmt;
 run;
```

fixed effects solution vector gives relations with W

```
 proc mixed data=yt;
   class case;
   model y = time W time*W / s;
   random int time / type=un sub=case gcorr;
   make 'SolutionF' out=solfout;
   %bystmt;
 run;
```

Raw SAS--- frames 7,8 NCFem; frame 7 Ramus; frames 7,8 Smearmiss;