

Overview and Implementation for Basic Longitudinal Data Analysis

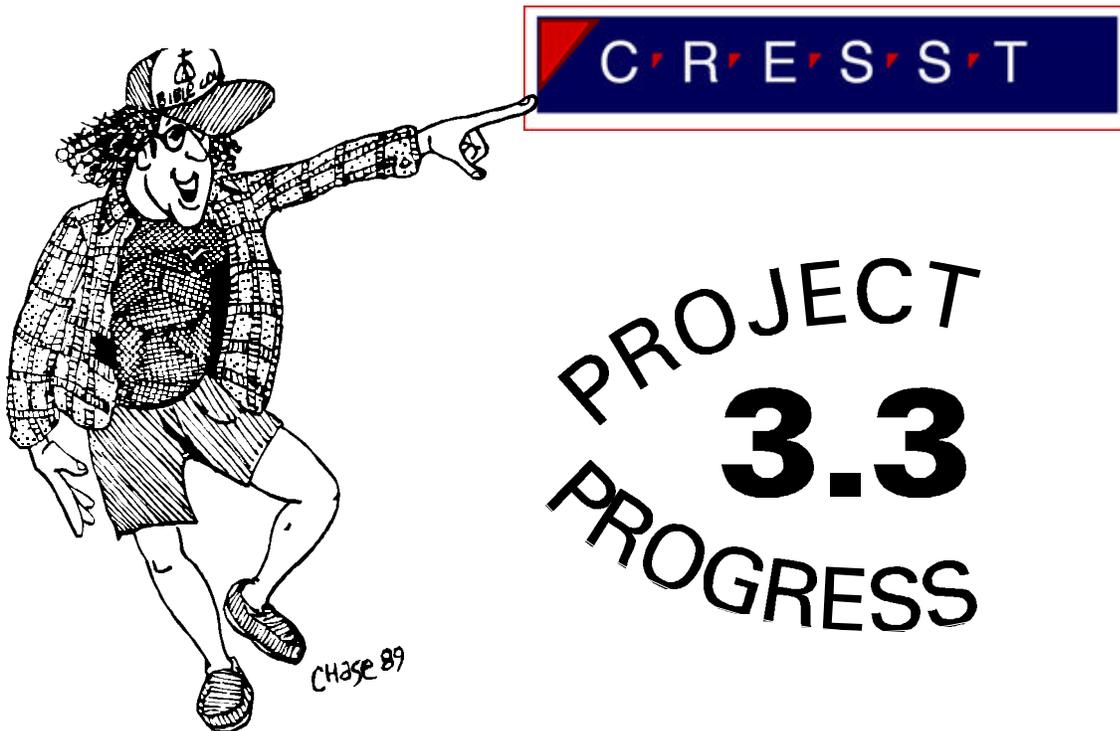
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MYTHS

about longitudinal research

Rogosa, D. R.

Myths about longitudinal research. Presented at the Stanford-Berkeley Colloquium on Quantitative Methods in Behavioral Science, U.C. Berkeley, May 1983.

Rogosa, D. R. (1988). Myths about longitudinal research. In *Methodological issues in aging research*, K. W. Schaie, R. T. Campbell, W. M. Meredith, and S. C. Rawlings, Eds. New York, Springer Publishing Company, 171-209.

Rogosa, D. R. (1995). Myths and methods: "Myths about longitudinal research," plus supplemental questions. In *The analysis of change*, J. M. Gottman, Ed. Hillsdale, New Jersey: Lawrence Erlbaum Associates, 3-65.

1. Two Observations a longitudinal study make.
2. The difference score is intrinsically unreliable and unfair
3. You can determine from the correlation matrix for the longitudinal data whether or not you are measuring the same thing over time
4. The correlation between change and initial status is:
(a) *negative*; (b) *zero*; (c) *positive*; (d) *all of the above*.
5. You can't avoid regression toward the mean
6. Residual change cures what ails the difference score
7. Analyses of covariance matrices inform about change
 - 7.1 Path analysis informs about change
 - 7.2 Structural regression models inform about change
 - 7.3 Simplex models describe most longitudinal data
8. Stability coefficients estimate:
(a) *the consistency over time of an individual*;
(b) *the consistency over time of an average individual*;
(c) *the consistency over time of individual differences*;
(d) *none of the above*; (e) *some of the above*.
9. Casual analyses support causal inferences about reciprocal effects

OLD BUSINESS

Conditional versus Unconditional Analyses (Goldstein, Plewis...)

[UK Reading example]

Longitudinal Data Examples

Individual Change Analyses

output sheets and data listings for some of the following examples:

Textbook Examples

Dental From: lme and nlme: Mixed-effects Methods and Classes for S and S-plus Jose C. Pinheiro, Douglas M. Bates. From an orthodontic study presented in Potthoff and Roy (1964). Four measurements of the distance (in millimeters) from the center of the pituitary to the pteryomaxillary fissure made at ages 8, 10, 12, and 14 years on 16 boys and 11 girls (gender used as exogenous W).

Ramus 4 longitudinal observations on each of 20 cases. The measurement is the height of the mandibular ramus bone (in mm) for boys each measured at 8, 8.5, 9, 9.5 years of age. used by a number of authors, can be found in Table 4.1 of Goldstein (1979)

Rat Rat weight data, from 1989 HLM manual. The rat data consist of 10 individuals, with weight measurements (Y) at 5 occasions (weeks 0,1,2,3,4) and a background measure (W), the mother's weight

Education Data Examples

WISC 4 observations, Wechsler Intelligence Scale for Children, Performance Scale, 86 children (times: begin first, end first, third, fifth grades). Gender W (Osborne).

NC Fem North Carolina Achievement Data (see Williamson, Applebaum, Epanchin, 1991). These education data are eight yearly observations on achievement test scores in math (Y), for 277 females each followed from grade 1 to grade 8, with a verbal ability background measure (W)

Canonical Artificial Examples (known structure)

Structure and Parameter Values for Artificial Data on p.6

ArtN200 used in Rogosa Saner 1995, both full data and Y-missing versions: 5 waves on each of 200 individuals, with observation times $\{0,1,2,3,4\}$, and with an exogenous variable W for each individual.

Smearmiss. Artificial longitudinal data with known structure. Five observations (about 16% missing) on each of 100 individuals, with times of observation varying over individuals, and with an exogenous measure W

Data Structures.

First five examples (Textbook, Education) have the simplest structure, no missing data, and "synchronous"--i.e., all individual measures at same times. In practice, data are missing; different observation times across individuals. Timepath97 estimation procedures for the general case.

For looking at Timepath97 and other estimation approaches, a

simple 2x2
Table is
useful:

		Outcome Observations	
		FULL	MISSING
Time Observations	SYNCHRONOUS	Rat, Ramus, WISC, Dental, NCFem	Rogosa-Saner Table 2
	NOT SYNCHRONOUS	Rogosa-Saner Reply (smear)	Smearmiss

Estimation: Full-Synchronous– closed-form mle; lisrel; OLS exact
Missing-NonSynchronous– mixed model (SAS PROC MIXED, S-plus lme, HLM); OLS approximate.

Models for Collections of Growth Curves

Straight-line Growth Curve Formulation.

attribute η , which exhibits systematic change over time. For individual p , growth curve in η is $\eta_p(t)$.

$$\eta_p(t) = \eta_p(0) + \theta_p t$$

Note: Rewrite using the centering parameter t^0 ; θ and $\eta(t^0)$ are uncorrelated over the population of individuals $t^0 = -\sigma_{\eta(0)\theta} / \sigma_\theta^2$

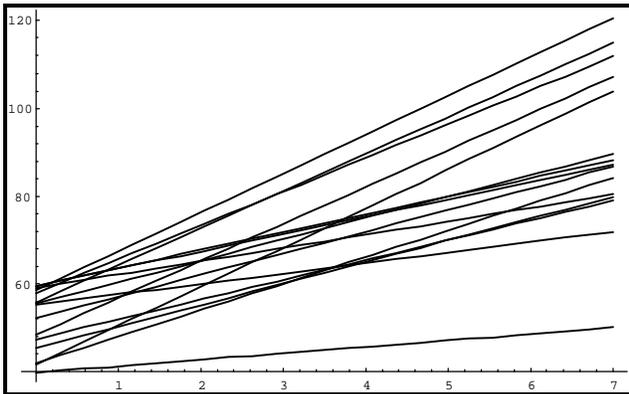
$$\eta_p(t) = \eta_p(t^0) + \theta_p(t - t^0) .$$

Constant rate of change θ_p -- first two moments $\mu_\theta \quad \sigma_\theta^2$

Shown below 15 straight-line growth curves

Straight-line Growth corresponding to pop. parameters $t^0 = 2$; $\sigma_\theta^2 = 5.333$; $\sigma_{\eta(t^0)}^2 = 48$; $\theta \sim U[1, 9]$, $\eta(t^0) \sim U[38, 62]$. correlations

among $\eta(t_i)$ for observation times $\rho_{\eta(1)\eta(4)} = .614$, $\rho_{\eta(1)\eta(6)} = .316$, $\rho_{\eta(4)\eta(6)} = .943$.



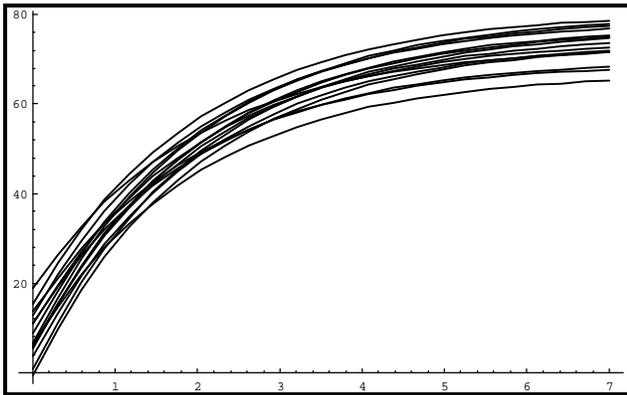
Observables. oversimplified version-- observable Y is an imperfectly measured η , relation between Y and η is simple classical test theory model: $Y_p(t_i) = \eta_p(t_i) + \epsilon_i$. For Y , with $\text{var}(\epsilon) = 5$, the pop. correlations are $\rho_{Y(1)Y(4)} = .567$, $\rho_{Y(1)Y(6)} = .297$, $\rho_{Y(4)Y(6)} = .894$.

For systematic individual differences in growth (i.e. correlates of change) exogenous characteristic W .

Conditional expectation $E(\theta|W) = \mu_\theta + \beta_{\theta W} (W - \mu_W)$. With no measured exogenous variable, this between-unit model is $E(\theta|W) = \mu_\theta$.

Alternative: exponential growth to an asymptote

Exponential growth curve with asymptote λ_p and curvature δ



$$\eta_p(t) = \lambda_p - (\lambda_p - \eta_p(0)) \exp(-\delta t)$$

Figure shows a collection of 15 exponential growth curves

exogenous variable W could be linked with both λ_p and $\eta_p(t^0)$.

Technical specifications for Artificial Data Structure.

The unobserved $\eta_p(t_i)$ follow the straight-line growth model, $\theta \sim N(5, 5)$; $\eta(0) \sim N(44, 52)$; $\eta(1) \sim N(49, 47)$; $\eta(2) \sim N(54, 52)$; $\eta(3) \sim N(59, 67)$; $\eta(4) \sim N(64, 92)$; an exogenous measure $W : W \sim N(10, 4)$; with the observables Y including the measurement error $\epsilon : \epsilon \sim N(0, 12)$. Model parameter values for the artificial data are $t^0 = 1$; $\sigma_\theta^2 = 5.0$; $\sigma_{\eta(t^0)}^2 = 47$ (yielding $\kappa = 3.066$); $\rho_{W\theta} = .60$; $\rho_{W\eta(1)} = .60$; for $\theta \sim N(5, 5)$, $\eta(t^0) \sim N(49, 47)$, $W \sim N(10, 4)$, $\epsilon \sim N(0, 12)$

Values of parameters of interest.

For examining the performance of the data analysis procedures we have population values determined as:

- a. rate $\mu_\theta = 5.0$ $\sigma_\theta^2 = 5.0$ $\rho(\hat{\theta}) = .806$. .
- b. Correlation between change and initial status. $\rho_{\eta(0)\theta}$ is $-.310$
- c. Regression for exogenous variable, $\beta_{\theta W} = .671$).

Potential observables Y at the observation times $t_i = \{0, 1, 2, 3, 4\}$ have population reliabilities $\{.813, .797, .813, .848, .885\}$

The upper triangle of the population correlation matrix for the $\eta(t_i)$ at the observation times $t_i = \{0, 1, 2, 3, 4\}$ is

+					,
*	0.951	0.808	0.627	0.463	*
*		0.951	0.838	0.715	*
*			0.966	0.896	*
*				0.981	*
.					-

longitudinal research questions and parameters of interest

for present purposes, analysis of individual change methods address the first three questions on my standard listing of research questions

1. Individual and Group Growth. Description of the form and amount of change, estimation of the individual (or group) growth curve, heterogeneity (individual differences) in the individual growth curves, and the statistical and psychometric properties of these estimates.

Parameters: $f(\theta; t)$, μ_θ σ_θ^2 $\rho(\hat{\theta})$ $\rho_{\eta(t)\theta}$

2. Correlates and Predictors of Change. systematic individual differences in growth e.g., "What kind of persons learn (grow) fastest?".

Parameters: $\rho_{\theta W}$ $\beta_{\theta W}$

3. Stability over Time. Questions about temporal stability fall into two broad headings--*Is an individual consistent over time? and Are individual differences consistent over time?* (Rogosa, Willett, Floden 1984) .

For consistency of individual differences over time:

Parameter: Foulkes-Davis $\gamma = \text{Pr}(\text{two growth curves do not intersect})$
"tracking" if index $> .50$ (significantly)

Common Claims: (empirical and metaphysical)

*everyone changes at the same rate (people interchangeable)

*change can't be measured reliably/accurately

*correlation of change and initial status is negative; regression toward mean pertains etc

Additional research questions

4. Comparing Experimental Groups.

5. Comparing Nonexperimental Groups.

(note: Dental, Wisc compare intact groups via W code)

6. Analysis of Reciprocal Effects.

7. Growth in Multiple Measures.

Theme for the day (and for the decades): Questions 1-3 addressed by growth-curve approach to the basic individual change problem.

This approach of modeling individual trajectories builds up to more complex settings such as-- group comparisons, hierarchical data structures, non-constant rate-of-change modelling etc.

Data Analysis and Parameter Estimation

Precursor: Descriptive Growth Curve Analyses

SFYS: fit Y on t regressions, describe resulting $\hat{\theta}_p$, fit $\hat{\theta}_p$ on W regr,
 Examples: WISC, frames 1-4; Ramus, frames 1-3; SmearMiss, frames 1-3.
 Even non-synchronous data, get variance comps and derived quants by
 approx method-of-moments (Rogosa-Saner 1995); works surprisingly well.

Maximum Likelihood estimation for parameters

Special, simple case; Complete, Synchronous Data.

ml estimation equations for full data in closed form (Blomqvist 1977)

example estimation of $\text{var}(\theta) \sigma_\theta^2$

MSR_p mean squared residual for OLS fit individual p ; $\hat{\sigma}^2$ is $\text{Ave}(MSR_p)$.

estimate for σ_θ^2 : $\hat{\sigma}_\theta^2 = \text{SS}(\hat{\theta}_p) / "n" - \hat{\sigma}^2 / \text{SSt}$,

reliability estimate for $\hat{\theta}_p$: $\hat{\rho}(\hat{\theta}) = \hat{\sigma}_\theta^2 / \text{SS}(\hat{\theta}_p) / "n"$

General strategy: get elements of 2x2 est. covariance matrix of θ and $\eta(0)$ for full or incomplete data. Common to **All programs** (LISREL HLM Tp) Tp: further substitute for derived quantities.

Also, fixed effects from separate run with W (when exists)–OLS equiv

properties of mle: bias, precision: Is reml best?

bias and mean-square-error : compare ML and REML

mle and reml simulation (50,000); complete synchronous data

		Estimation of σ_θ^2 var(theta) = 5.0	
		ML	REML
n			
10	4.37 [7.39]	4.99 [8.61]	
15	4.58 [5.06]	4.99 [5.59]	

MAJOR MESSAGES

1. OLS equivalences for fixed effects; Method-of-moments match for random effects
2. 2x2 covariance matrix ($\eta_p(0) \theta_p$)-- elements σ_θ^2 $\sigma_{\eta(0)}$ $\sigma_{\eta(0)\theta}$ --starting point for growth statistics
3. uncertainty, via s.e. and CI, reporting essential--for small (or medium) n, BCa intervals vs standard

From Growth Curves to Mixed(Random)-Effects Models

$$N_p(t) = N_p(0) + \sigma_p t$$
 Data
$$Y_{pj} = N_p(t_{pj}) + \epsilon_{pj}$$
 Intercepts and slopes may differ across $p = 1, \dots, N$
 ordered times $j = 1 \dots T_p$ obs for p (missing or not)

No ω	Random	Fixed
$E_p(\sigma_p) = \mu_\sigma$	$N_p(0) - \mu_{N(0)}$	$\mu_{N(0)}$
$E_p(N_p(0)) = \mu_{N(0)}$	$\sigma_p - \mu_\sigma$	μ_σ

Mixed Effects Model

$$Y = X\beta + Z\gamma + \epsilon$$

$N \times \sum_{p=1}^{T_p} \times 1$ $\sum_{T_p} \times 2 \quad 2 \times 1$ $\sum_{T_p} \times 2N \quad 2N \times 1$

fixed avg. growth curve

$$X = \begin{bmatrix} 1 & t_{p1} \\ \vdots & \vdots \\ 1 & t_{pT} \end{bmatrix}$$

$$\beta = \begin{bmatrix} \mu_{N(0)} \\ \mu_\sigma \end{bmatrix}$$

random

$$\gamma = \begin{bmatrix} N_p(0) - \mu_{N(0)} \\ \sigma_p - \mu_\sigma \end{bmatrix}$$

2 rows each p

blocks

$$Z = \begin{bmatrix} \begin{bmatrix} 1 & t_{p1} \\ \vdots & \vdots \\ 1 & t_{pT} \end{bmatrix} & \dots & \begin{bmatrix} \end{bmatrix} \end{bmatrix}$$

block elements of G

$$\text{Var}(Y) = V = ZGZ' + R$$

$$G = \begin{bmatrix} \sigma_{N(0)}^2 & \sigma_{N(0)\sigma} \\ \sigma_{N(0)\sigma} & \sigma_\sigma^2 \end{bmatrix}$$

also D aka

From Growth Curves to Mixed(Random)-Effects Models

with w

Fixed

$$E(\eta_i | w) = \mu_{\eta_i} + \beta_{\eta_i} w (w - \mu_w)$$

$$E(\sigma_i | w) = \mu_{\sigma} + \beta_{\sigma} w (w - \mu_w)$$

Random

$$\eta_p(w) - E(\eta_i | w)$$

$$\sigma_p - E(\sigma_i | w)$$

$$Y = X\beta + Z\gamma + \epsilon$$

expanding X
 $\sum T_p \times 4$

fixed

$$= \begin{bmatrix} 1 & t_{p1} & w_p & w_p * t_{p1} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & t_{pT_p} & w_p & w_p * t_{pT_p} \end{bmatrix}$$

$$\beta = \begin{bmatrix} \mu_{\eta_i} - \beta_{\eta_i} \mu_w \\ \mu_{\sigma} - \beta_{\sigma} \mu_w \\ \beta_{\eta_i} w \\ \beta_{\sigma} w \end{bmatrix}$$

int
time
 w
time $\times w$

random

$$\gamma = \begin{bmatrix} \eta_p(w) - E(\eta_i | w_p) \\ \sigma_p - E(\sigma_i | w_p) \\ \vdots \end{bmatrix}$$

$2N \times 1$

G-matrix contains conditional variances

$$\text{Var}(\sigma | w) \quad \text{Var}(\eta_i | w)$$

see NCFem, Frames 7, 8 etc

Timepath97



A dissemination project

Timepath97-parameter estimation

- * obtain estimates for growth curve quantities of interest from solutions (using Make, ODS facility for 6.11+) estimated covariance parameters give t^0 , κ , variances and derived quantities (see page 12) etc;
- * embed in jackboot.sas to obtain BCa confidence intervals for derived quantities (link to jackboot and docs on Timepath97 Web page)

Implementation of Estimation using SAS- PROC MIXED

(thanks to Neil Timm, Univ Pitt. & Russ Wolfinger, SAS Inc)

REML, ML etc available. (REML matches other E-M programs, e.g SmearMiss via HLM).

S-plus Alternative: lme-- Pinheiro & Bates, or further with nlme

<http://netlib.bell-labs.com/cm/ms/departments/sia/project/nlme/index.html>

put data in column form [ID, Y, t, W] Run PROC MIXED without and with W to obtain core quantities for parameter estimation

From no-W run obtain Covariance Parameter Matrix (G);

```
/* Proc mixed run */
proc mixed data=yt;
  class case;
  model y = time / s;
  random int time / type=un sub=case gcorr;
  make 'CovParms' out=untot;
  make 'SolutionF' out=solfout;
  %bystmt;
run;
```

fixed effects solution vector gives relations with W

```
proc mixed data=yt;
  class case;
  model y = time W time*W / s;
  random int time / type=un sub=case gcorr;
  make 'SolutionF' out=solfout;
  %bystmt;
run;
```

Raw SAS--- frames 7,8 NCFem; frame 7 Ramus; frames 7,8 Smearmiss;

What to do with covariance matrix ($\eta_p(0)$ θ_p) ?

Timepath97 Output, in each data example, constructed from SAS (reml or ml) core estimates

Extensions using properties of collections of growth curves

To estimate growth-curve quantities of interest, substitute core estimates into relations among moments: selected given below

variance

$$\sigma_{\eta(t)}^2 = \sigma_{\eta(t^0)}^2 + ((t - t^0)/\kappa)^2 \sigma_{\eta(t^0)}^2$$

covariance (also yields correlation, using above)

$$\sigma_{\eta(t_1)\eta(t_2)} = \sigma_{\eta(t^0)}^2 + (t_1 - t^0)(t_2 - t^0) \sigma_{\theta}^2$$

correlation between change and status

$$\rho_{\eta(t)\theta} = \frac{(t - t^0)}{[\kappa^2 + (t - t^0)^2]^{1/2}}$$

correlation between exogenous variable, W and status

$$\rho_{W\eta(t)} = \frac{(t - t^0)\rho_{W\theta} + \kappa \rho_{W\eta(t^0)}}{[\kappa^2 + (t - t^0)^2]^{1/2}}$$

ASSESSMENTS OF STABILITY: index of tracking Foulkes and Davis (1981): "tracking" if index > .50 (significantly)

Estimation. Fit individual trajectories (straight-line or polynomial etc).

For each individual compute the proportion of other trajectories not crossed.

Point estimate is the average over individuals of these proportions.

F-D p.441 use standard deviation of individual estimates divided by Sqrt[n] as the standard error and construct normal theory CI.

Bootstrap results array, in each data example, constructed by reformatting output from jackboot.sas. Choose quantities to bootstrap...

Examples: SmearMiss parameter est.; Ramus frame 6.

Section 1- Descriptive analyses of growth rates.**Individual OLS Fits:**

individual fits of a straight-line growth curve (the regression of Y on t for each p) by ordinary least-squares.

Cross-sectional Description: For *synchronous* data sets

OLS Fits: Descriptive Statistics:

comparisons of rates of change across individuals. Stem-and-leaf diagrams and accompanying boxplots for RATE (Empirical rate) INIT_LVL (Fitted Initial Level) W (Exogenous Variable).

OLS Fitted Values for Anchor Times

Rate and Fitted Initial Level: Scatter plot of RATE vs INIT_LVL

OLS Theta-hat on W Regression: When W is present, OLS regression and corresponding scatterplot is given—provides graphical diagnostic and shows correspondence (both point estimate and standard error) with fixed-effects estimates from mixed-model estimation (exact match for complete synchronous data)

OLS Fitted Initial Level on W Regression: When W is present, OLS regression is given—provides graphical diagnostic and shows correspondence (both point estimate and standard error) with fixed-effects estimates from mixed-model estimation (exact match for complete synchronous data)

Section 2- Parameter Estimates

Parameter Estimates collection of estimates based on the growth curve model. First parameters listed are "typical" rates of change μ_θ or median(θ), and a measure of heterogeneity σ_θ^2 , the variance of the θ_p . Then estimate of the reliability $\rho(\hat{\theta})$. For each of the stated anchor times, the quantities estimated are: $\sigma_{\eta(t)}^2$, $\rho_{\eta(t)\theta}$, $\beta_{\theta\eta(t)}$ and Reliability $Y(t)$.

Using the chosen anchor time point, estimate of $\rho_{\eta(t)\theta}$;

The index of tracking γ (Foulkes & Davis 1981);

Systematic individual differences in growth $\rho_{\theta W}$ or by $\beta_{\theta W}$;

Section 3-Inference using Bootstrap Resampling**Bootstrap Confidence Intervals**

Bootstrap estimation is provided for the following parameters

μ_θ Mean(Rate); Median(θ); σ_θ^2 var(Rate); $\rho(\hat{\theta})$ rel(Rate);

$\rho_{\eta(t)\theta}$ Corr(Rate, Initial status); $\beta_{\theta W}$ Beta(Rate, W)

$\beta_{\eta(t)W}$ Beta(Eta(T1), W)

Notes on Data Analysis Examples

OLS equivs for ramus, artN200 (see auxiliary sheet) approx smear
Fixed Effects Smearmiss output

SAS Solution for Fixed Effects

Effect	Estimate	Std Error	t	Pr > t
INTERCEPT	30.47409042	3.83950806	7.94	0.0001
TIME	-0.92464420	1.18313096	-0.78	0.4364
W	1.29057304	0.37026530	3.49	0.0006
TIME*W	0.60101799	0.11408524	5.27	0.0001

OLS output in Timepath97

Simple OLS Theta-hat on W Regression

Variable	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	-0.901105	1.16435047	-0.774	0.4408
W	0.594518	0.11248947	5.285	0.0001

Simple OLS Fitted Initial Level on W Regression

Variable	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T
INTERCEP	30.601655	3.80705611	8.038	0.0001
W	1.286608	0.36780483	3.498	0.0007

Bootstrap output in Timepath97 (on auxiliary sheet)

Importance of Standard errors small n, estimation imprecise (most often).

advantages of BCa (or ABC) intervals

RAT (HLM n=10)

* var(theta) *

\$Desc:	mle	mean.boot	se.boot	se.SAS
	10.53901	11.86508	7.613102	9.59
\$CI:	0.025	0.05	0.95	0.975
BCa	1.2796840	1.751531	27.27126	31.87409

* rel (thetahat) *

\$Desc:	mle	mean.boot	se.boot
	0.534604	0.5142543	0.1724859
\$CI:	0.025	0.05	0.95
BCa	0.1255408	0.1753536	0.7408227

* corr trchange,initial status *

\$Desc:	mle	mean.boot	se.boot
	-0.1227116	-0.2241844	0.418065
\$CI:	0.025	0.05	0.95
BCa	-0.7646482	-0.6387207	0.8164504

Dental (n=27) rel(Rate) in {0,.8}, Corr(Rate, Initial Status) in {-.4,1.0}

Ramus (n=20) does better than Dental

Other Lessons: Corr(Rate, Initial Status) can be positive and large.

examples: NCFem, WISC Change can be assessed accurately, reliably

Further Investigations

LISREL equivalence adventure–Joreskog vs Gauss?

for Full, Synchronous data LISREL can be formulated (following roughly Willett-Sayer) to carry out a (reml) growth curve estimation. LISREL works almost as well as OLS. See attached model formulation and output (ver 8). In ArtN200 example results reasonable match reml estimates (except for s.e. of some fixed effects). Fit statistic?

Some extensions possible with LISREL that cannot be accomodated with other approaches.

AR(1) Excursion (with ArtN200)

1. AR(1) data construction

for $\eta_p(t)$ from ArtN200 construction add AR(1) $\rho = .3$ errors $e \sim N(0,12)$

2. Effects of AR(1),

SAS

AR(1) Errors		iid Errors	
Cov Parm	Estimate	Cov Parm	Estimate
UN(1,1)	62.74926568	UN(1,1)	55.11544454
UN(2,1)	-6.75750327	UN(2,1)	-3.80814129
UN(2,2)	5.33756422	UN(2,2)	4.39372614
Residual	8.72009620	Residual	11.90899971

LISREL (model mod) similar values to SAS AR(1)

MAXIMUM MODIFICATION INDEX IS 9.86 FOR ELEMENT (5, 1) OF THETA-EPS

3. Amelioration–fitting AR(1) error structure

SAS line for PROC MIXED /* put in AR(1) */

repeated /type=AR(1) sub=case;

UN(1,1)	60.16344992
UN(2,1)	-6.04826235
UN(2,2)	4.92409041
AR(1)	0.24049755
Residual	11.05217550

LISREL more dicey to set up constraints–starting values problems, better way to do AR(1)? Band diag works pretty well

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