Stat 315c: Introduction

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Stanford Statistics
### Usual Statistics Setup

- there’s $Y$ (we’ll predict it)
- and there’s $X_1, \ldots, X_d$ (to predict from)
- and $n$ IID copies of $(X, Y)$ to infer with
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**Data matrix $(X, Y)$ is $n$ by $d + 1$**

- $d + 1$ named columns (variables)
- and $n$ anonymous exchangeable rows
- with $n \to \infty$ and $d$ fixed
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### This course

- Both rows and columns are named
- We learn about the cols using rows as obs, and conversely
- $n$ and $d$ may both be large
## Two mode examples

- **Movies × Raters → Ratings**
- **Terms × Documents → Counts**
- **Genes × Experiments → Expression level**
- **IP-address × Books → Purchases**
- **Questions × Test takers → Grade**
Problem domains

Two mode examples

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Single mode examples (Rows × Cols are the same entities)

- Actors × Actors → co-appearance
- Articles × Articles → co-citation
- Web pages × Web pages → hyper-links
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Higher dimensional layouts

- genes × conditions × tissues
About the course

History
- Response to common thread in lots of problems
- Began as seminar in spring 2000
- Guest speakers from Netflix and biomedical informatics and statistics

Goals
- Look at existing methods
- Look for remaining holes

Materials
- Articles online
- There’s no book
Data types

**Dyadic data**

For $X \in \mathcal{X}$ (eg actors) and $Y \in \mathcal{Y}$ (eg movies)  
Record pairs 

$$(X_1, Y_1), (X_2, Y_2), (X_3, Y_3), \ldots (X_N, Y_N)$$

Actor $X_i$ was in movie $Y_i$  
$N \ll |\mathcal{X}| \times |\mathcal{Y}|$ so the full matrix would be very sparse

So it is “variables and cases as usual”, after all

- Variable 1 = actor, Variable 2 = movie
- maybe Variable 3 = box office
- $N = \# \text{ pairs} \to \infty$, with $d = 2$ or $3$

(Well almost)
Back to anonymous rows, but with

A special kind of random variable

- Categorical with many levels, e.g.:
  1. Phone number
  2. IP address
  3. Actor
  4. Query string

- Number of levels grows with $N$

- There may be many unseen levels
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Different from classical categorical variables, e.g.:

- Binary variables, or,
- Setosa vs Virginica vs Versicolor, etc.
Non-dyadic examples

Dense data

- for microarrays we have all genes in all experiments apart from missing values
- for dyadic case, it’s mostly missing apart from a few observed values

Triadic data

- Actors, Directors, and Year
- Genes, Conditions, Tissues
Graphs

Transposable data often have a graph representation.

- Edges can be directed or undirected
- Bipartite graphs for data of two modes (e.g., actors and films)
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Edges can have weights (more generally feature vectors)
- Nodes can have features
- Hypergraphs for triadic data
- Generalize ad infinitum (but then we break the graph paradigm)
Methods

Methods for these problems are of several (overlapping) types

- **Classical**
  1. ANOVA
  2. Correspondence analysis
  3. Rasch model
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- Unsupervised learning
  1. Clustering (group the rows or the columns)
  2. Biclustering (jointly group the rows and the columns)
  3. Spectral clustering
  4. Independent components analysis
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- **Matrix approximation**
  - 1. Singular value decomposition
  - 2. Nonnegative decomposition
  - 3. Semi-Discrete decomposition
But wait there’s more

Some more ideas, not yet forced into a category

- PageRank, TrustRank, Hubs and Authorities
- Smoothing on graphs
- Subsampling matrices
- Recommender engines
- Archetypal analysis
- Latent Dirichlet Allocation
- Compositional data
- Canonical correlation and generalizations
- Head versus long tail
Problems and tasks

We’d like to

- Predict missing labels (eg spam)
- Find anomalies (eg unusual credit card patterns)
- Decide where to get labels
- Group rows/columns/both
- Reduce dimension
- Predict missing links

Goals

- Find common structures in these problem
- Learn some specific methods
- Learn to compare|mix|hybridize methods
- Move from “could to” to “should do”
- Spot research opportunities
High level view

Approaches include

- Principled Bayesian methods
- Ad hoc but very fast algorithms
- Moments
- Maximum likelihood

Persistent issues

- What happens to the bootstrap and cross-validation?
- How should we window data arriving in time?
- Does anything go to \( \infty \)?
- Do we model missingness?

When we're done

There will be lots of holes in the material

Right now there are disconnected islands
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Not a usual course
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Notice:

-Ernest Shackleton-
Not a usual course

Then

- It was 1914
- 5000 people applied

Now

- Men and women wanted
- It won’t be cold

Notice:
Men wanted for hazardous journey.
Small wages. Bitter cold.
Long months of complete darkness.
Constant danger. Safe return doubtful.
Honour and recognition in case of success.

-Ernest Shackleton-
Some results

- New cross-validation method for (Perry and O.)
- New bootstrap for non-IID data (O.)
- Two papers on spectral clustering (Salzman)

From 07/08