1. Lookback option

A lookback feature in an option guarantees you the right to buy low (or to sell high). Suppose that a stock takes values $X_0, X_1, X_2, \ldots, X_T$ at equispaced times 0 through $T$. The present is time 0. Suppose that at time $T$ the (buy low) lookback option pays

$$X_T - \min_{i \in \{0,1,2,\ldots,T\}} X_i,$$  \hspace{1cm} (1)

and that at time $T$ the (sell high) lookback option pays

$$\max_{i \in \{0,1,2,\ldots,T\}} X_i - X_T.$$  \hspace{1cm} (2)

To simulate one path of stock prices, take

$$X_i = X_{i-1} \exp \left[ (r - 0.5\sigma^2)h + \sigma Z_i \sqrt{h} \right], \quad 1 \leq i \leq T;$$  \hspace{1cm} (3)

where $r$ is the risk neutral return, $\sigma^2$ is the volatility, $h$ is the number of years elapsing between consecutive time points ($i$ and $i + 1$), $Z_1, \ldots, Z_T$ are independent $N(0, 1)$ random variables (Matlab command RANDN), and $X_0$ is the known starting price.

[Equation 9.4.5 in the text is similar to (3) except that it omits the $-0.5\sigma^2$ term. The text by Hull gives both versions, but indicates that equation (3) is more accurate.]

(a) Take $T = 12$, $h = 1/12$, $r = 0.06$, $\sigma = 0.2$, and $X_0 = 100$. Plot 10 random price trajectories of the underlying stock.

(b) Produce a histogram of 1,000 independent random payoffs of a lookback buy-low option with the parameters in part a.

(c) Produce a histogram of 1,000 independent random payoffs of a lookback sell-high option with the parameters in part a. Use the same random price trajectories as in part b. (That is, save your $Z$ values for reuse.)

(d) Plot the buy-low payoff versus the sell-high payoff for the 1000 simulations.
(e) What is the Monte Carlo value of these options? (Take the average payoff, and multiply it by a discount factor $\exp(-\tau \gamma)$ where $\tau$ is the number of years that elapses between the present and the payoff.)

(f) Using the same random price trajectories, investigate the effect of $\sigma$ on the value of these options. Find the value for $\sigma = .02$ to $.40$ by steps of $.02$. [Using the same random numbers at all $\sigma$ values makes these plots much clearer.]

(g) For the options in part a, suppose it is desired to price them with a Monte Carlo standard error of 0.1. Using the 1000 values you have, estimate how large the number $n$ of simulations required for this accuracy would be.