A NUMERICAL ANALYSIS OF COORDINATING MONETARY POLICY UNDER NEW KEYNESIAN MACROECONOMICS: CAN IT BE ACCOMPLISHED, HOW BIG ARE THE EFFECTS, AND WHY?

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ABSTRACT

This paper tests whether or not gains to monetary policy coordination exist in the New Keynesian two country model proposed in Clarida, Gali and Gertler (2002). Major innovations include rigorous numerical analysis that takes advantage of recent techniques proposed by Laffargue (1990) and Klein (2000) and an attention to robustness analysis as advocated by McCallum (1999) by comparing all results to the rational expectations model proposed in Taylor (1980). The major results found are (1) any gains from coordination in Clarida, Gali and Gertler (2002) are not statistically significant, (2) a “simple rule” approach to monetary policy produces policy rules that outperform policy based on discretionary assumptions, regardless of the decision to coordination, and (3) the model in Clarida, Gali and Gertler (2002) can be characterized by a distinct lack of open economy interaction.

Keywords: monetary policy coordination, New Keynesian, open economy, nominal rigidities, discretionary policy, deterministic simulation, stochastic simulation, linearized general equilibrium dynamic models

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I. INTRODUCTION

Anyone would be hard pressed to argue against increasing global integration. Yet the effects of integration, and its celebrity-like cousin, globalization, on international economies are much more controversial.

A particular contribution to the subject has been the concept of monetary policy coordination, or the idea of overall welfare benefits when the central banks of multiple countries work together in forming a globally optimal monetary policy instead of only maximizing welfare within the boundaries of the domestic economy. In short, what is the pareto-optimal solution to international monetary policy?

This paper adds to the literature on monetary policy coordination by numerically testing whether or not gains to coordination exist in the New Keynesian, two country model proposed in Clarida, Gali and Gertler (2002). The approach is unique in that it focuses on one main consideration that the author feels has not been adequately addressed in the current literature – robustness analysis.

As discussed in McCallum (1999), any model of the economy, or anything else for that matter, is just that, a model. At best it will approximate some facet of the real world reasonably well. At worst, it will do so extremely poorly. No matter how accurate your model is in its predictions, it will never be free from criticism. “King (1993) and Fuhrer (1997) would point to weaknesses in modeling investment or consumption behavior.” (McCallum, 1999) Corsetti and Pesenti (2001) may focus on the relation of exchange rates to price setting behavior, while Clarida, Gali and Gertler (2002) would probably emphasize nominal rigidities. Thus, given that it is inherently impossible to build a model that everyone can agree on, and that any single model is an imperfect representation of the world, it is important to attempt to produce results that are robust to a variety of different models.

This paper adheres to this logic in the sense that every result is verified against the rational expectations model presented in Taylor (1980). The reasons for comparing against this model are many
and are detailed over the following chapters. The benefit is that, besides achieving more convincing results, I am able to exploit similarities and differences between the models in order to describe the economic intuition behind my results to an extent that neither model would offer by itself.

The major conclusions are that there are no gains from coordinating monetary policy. Although theoretically there may be benefits to coordination in the Clarida, Gali and Gertler model, numerical analysis shows that these benefits are not statistically significant. Second, there exist outside policies that outperform both the coordinated and non-coordinated monetary policies considered in this paper. This is due primarily to the assumption that central banks operate under a discretionary regime. Relaxing this assumption produces significant gains in optimizing policy. And third, the Clarida, Gali and Gertler (2002) model can be characterized by a distinct lack of interaction in the open economy. This, in turn, may explain why it is impossible to find welfare gains to coordination. The size of international effects are so small that it really does not matter which policy the foreign economy chooses to follow.

The first two conclusions are fully supported by results of the same tests in the Taylor (1980) model. The third, in turn, is helped to be explained by it. In order to arrive at similar equations between the two models, the dependence of prices on the foreign variables have to be turned off in Taylor (1980). This suggests that a significant characteristic of the open economy in Taylor (1980) is not present in Clarida, Gali and Gertler (2002).

The next section is a review of the literature, which covers a history of coordinating monetary policy from its early stages to contemporary techniques in numerical analysis. Section 3 introduces the models and describes the major similarities and differences between them. Section 4 traces through the economics behind an impact of similar exogenous shocks to each model. This is to put in context the analysis of subsequent sections. Section 5 numerically analyzes open economy effects under the Clarida, Gali and Gertler model. Section 6 introduces the simple rule policy and contrasts it to the
discretionary policy assumed for coordination. Section 7 reviews the results of optimal policy and coordination experiments, while section 8 checks the robustness of the results under Taylor (1980). Last but not least, section 8 concludes.

II. LITERATURE REVIEW

II.1 Monetary Policy Coordination – A Brief History

One of the earliest publications to discuss the coordination of monetary policy post World War II was a Brookings Institute publication in 1982. The paper defines coordination as using monetary policy “to stabilize nominal or real exchange rates or for targeting monetary policy on the nominal exchange rate” (Branson, 1982) and subsequently recommends against it. Although Branson showed that fluctuations in the real exchange rate could cause fluctuations in US employment, particularly in the durable goods sectors, and, hence, there is at least rational for worrying about stabilizing exchange rate fluctuations, he concludes that there is just too much uncertainty over the analytical causes of exchange rate fluctuations and too little international cohesion among economic powers to make coordination a viable option.¹ In Branson's own words, “let the central banks do the coordination and the National Science Foundations of the world finance research on the analysis.” (Branson, 1982)

Through the early 1990's, not much research was conducted in analyzing coordinated policy. One of the subsequent results came as a consequence of the 7 country, macro model built in Taylor (1993). It will be useful to summarize the Taylor (1993) model both for justification of its contribution to the policy coordination question and as an example of the macro model prevalent at the time – a macro-based, rational expectations model with price rigidity.

¹ Branson's findings would doubtfully be very different today (cite paper that finds exchange rate is essentially a random walk).
In his book, Taylor constructs a medium scale rational expectations econometric model of the G-7 countries: Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States. It harbors an impressive 98 stochastic equations and identities to model both the domestic economies of all 7 countries as well as the international interactions between them all. Perhaps the model's most prominent defining characteristic is the structure of wages as staggered contracts in the economy. This characteristic not only gives the model nominal rigidity, but also a microeconomic definition for why rigidities occur. In this sense, the Taylor (1993) model can be thought of as a bridge between more traditional Mundell – Fleming models and the completely micro-founded macro models of the New Keynesian literature in common practice today.²

Other notable characteristics of the Taylor (1993) model include that interests rates and exchange rates are determined in a worldwide capital market where capital flows freely between countries. Specifically, the interest rate differential between any two countries is determined by the expected change in exchange rates. The model incorporates consumption smoothing as well as slowly adjusting import prices and import demand. Also, the parameters of the model are completely estimated by regression on quarterly macroeconomic data from the 1970s and 80s. We can consider the Taylor (1993) model to be one of the more sophisticated and realistic models of its time.

In Chapter 6, Taylor runs a hypothetical policy experiment to answer the question of whether countries would ever want to coordinate in setting the coefficients in their policy rule. He considers nominal-income rules of the form:

\[
\begin{align*}
RS_i - RS_i^0 &= LP_i(4) - LP_i + g(LP_i - LP_i^0) + g(LY_i - LY_i^0) \\
\end{align*}
\]  

(2.01)

where \( RS \) is the real interest rate, \( LP \) is the log of price level, \( LY \) is the log of real income, \( g \) is a

² Incidentally, clarifying this relation becomes one of the objectives of this paper, and is the topic of section (x.x)
coefficient to be determined, and a superscript 0 denotes the target rate. Assuming that the objective of monetary policy is to minimize deviation of output and inflation from their long run trends, Taylor then analyzes the effect of one country changing the country specific coefficient g on the variation of output and inflation in the other 6. Here variation of output and inflation is defined by the root mean squared percentage deviation of the variable from trend.

Varying the coefficient g in a band of reasonable values does not have a significant effect on the variation of output and inflation in foreign economies. Taylor also finds similar results when the policy rule considered is a price rule without income terms. In fact, the model exhibits a general international insensitivity to monetary shocks. Hence Taylor concludes that gains from coordinating monetary policy would be minimal, if present at all.

The past decade has seen a marked rise in monetary policy coordination literature. Works such as Benigno and Benigno (2001), Clarida Gali and Gerler (2002), Sutherland (2004) and Michaelis (2006) have not only focused specifically on the coordination topic, but, perhaps more surprisingly, have all found instances where gains from coordination are possible. One common thread cuts across all of these works – they incorporate New Keynesian stochastic models into their analysis. I provide a history of these micro-founded macro models and describe the current field of coordination literature in their context.
II.2 Micro-Founded Macro Models

The commonly accepted Mundell – Fleming models, which largely disregarded the decisions made by individual consumers and producers, have been updated to incorporate microeconomic elements into macroeconomic policy analysis.

The effect of this shift in the approach to modeling open economies has been two fold. First, it allows for explicit calculations of profit and utility maximizing equations. Having hard numbers to back up policy analysis brings welcome clarity and precision. This, in turn, gives economists an opportunity to debate complex questions in a credible and objective light. Second, modeling the individuals and sectors within economies leads to aggregate welfare calculations. This brings a uniform metric to the table for comparing policy and assessing the impacts of various economic characteristics.

In more recent models, the general trend has been a systematic relaxation of simplifying assumptions (unitary elasticities, linear utility models, and limited financial markets are a few examples) to get at an ever more accurate representation of the international economy. Many works include stochastic modeling to bring uncertainty into the analysis as well.

II.2.1 The Model that Started it All – Obstfeld and Rogoff³

Obstfeld and Rogoff (1995) describe a model of two, large, open economy countries. Each country consists of two sectors: individual farmers, who at once represent both consumers and producers, and the government, responsible for changing the supply of money and giving out monetary transfers.

Of the individuals, a proportion, n, live in the home country and the rest, 1-n, live in the foreign

³ This section borrows from the exceptional survey of the literature in Lane (2001).
country. Each individual produces a unique good. The preference function for a single individual, j, across all periods of time (…, t-1, t, t+1, …) is given by a summation of preferences over an index that aggregates across all consumption goods (both home and foreign), money supply, and a negative preference for work. The price index in the model is given by a similar aggregation of prices over all possible goods.

The Obstfeld Rogoff (1995) model assumes zero government consumption. Hence, anything earned from the creation of money is returned to the economy by means of government transfers.

Finally, the model introduces an international real bond market with no risk and constant interest rate, r. Our farmer j’s budget constraint becomes a function of present and future holdings of money, bonds, the good he produces and consumes, and any transfers offered by the government.

The model assumes that all individuals have identical preferences and there are no barriers to trade. Hence, purchasing power parity holds and the real exchange rate is always held constant. Prices are exogenously sticky by being assumed to be set one period in advance (i.e. period t+1). This allows analysis of the model both in the short run (the period directly after a shock when prices have not had a chance to adjust; period t+1) and the long run (everything after; periods t+2, t+3, …).

One of the biggest contributions of Obstfeld Rogoff has been to demonstrate the benefits of using a microfounded model. For example, by viewing the utility of a representative agent, it is possible to show that both countries gain equal utility from an unexpected domestic monetary expansion (Lane, 2001).

II.2.2 Adapting Obstfeld Rogoff for Monetary Policy Coordination

With specific welfare calculations across two countries and an avenue for governments to follow and implement monetary policy, the model above becomes a useful to access monetary policy
coordination. Clarida, Gali and Gertler (2002) provide one such method for evaluation.

According to the authors, choosing to coordinate or not is grounded in the goals of policy makers. A country that chooses not to coordinate maximizes the utility of the domestic consumer, subject to the constraints posed by the formulas defined in the model, while taking all foreign variables as given:

\[
\text{Max } U(C_t) - V(N_t) \quad \text{st} \quad C_t = k(N_t)^{1-\gamma}(Y^f)^\gamma. \quad (2.02)
\]

In the above equation, \( U(C_t) \) is the period \( t \) utility from consumption while \( V(N_t) \) is a general value function representing loss of utility from labor. The foreign output, \( Y^f \), is taken to be constant under non-cooperative policy.\(^4\)

Clarida, Gali and Gertler (2002) then show that the second order Taylor expansion of the maximization quantity, taken about the levels of consumption and labor that prevail under perfectly flexible prices, is approximately equal to:

\[
U(C_t) - V(N_t) \approx D[\xi \sigma^2_{p,t} + \kappa \gamma^2_t] + o(||a||^3) \quad (2.03)
\]

In the above equation, \( D \) depends on consumption at time \( t \), \( \sigma^2_{p,t} \) reflects the cross-sectional dispersion of prices, and \( o(||a||^3) \) are all higher order terms. Finally, a result from Woodford (2001) can be used to express the above equation as a sum over all time and equate the dispersion of prices, \( \sigma^2_{p,t} \), to the square of inflation, \( \pi_t^2 \). Rearranging terms and substitution gives the following

\(^4\) This definition differs slightly from the commonly accepted theory that, under a Nash equilibrium, a non-coordinating country would assume that the foreign country behaves in a similarly optimal manner. Hence instead of assuming \( Y^f \) is constant in the above equation, the optimal choice of \( Y^f \) would be substituted in directly. An interesting question for further research would be how this change in definition affects the resulting optimal policy.
second order approximation to the objective of the domestic central bank:

$$W_{NC} = -\frac{(1-\gamma)}{2} \Lambda E_0 \left[ \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \alpha y_t^2) \right] \quad (2.04)$$

where $\Lambda = \frac{\bar{E}}{\delta}$, $\alpha = \lambda / \xi$, $\xi$ is the price elasticity of demand for intermediate goods, and $\lambda$ is the elasticity of inflation to changes in the domestic output gap from trend levels. The foreign objective function is completely isomorphic with $(1-\gamma)$, the relative weight of the domestic country replaced by $\gamma$, the weight of the foreign country, and all coefficients pertaining to the foreign country. Note the objective function, (2.04), is strikingly similar to the objectives commonly assumed for central banks without explicit welfare calculations. This gives further weight to the argument that a primary goal for monetary policy should be to control variation in inflation and output, as well as allows for a measure of continuity between past and contemporary studies.

The second case, of counties choosing to coordinate monetary policy, assumes there is a central planner whose objective is to maximize global utility given by

$$U(C_t) - (1-\gamma)V(N_t) - \gamma V(N_t^f) \quad (2.05)$$

Now $C_t$ denotes global consumption, and is constrained according to $C = (N)^{1-\gamma} (N^f)^\gamma$. Instead of each country maximizing their own objective function, isolated from the rest of the world, the goal is to consider both countries in tandem. The weight placed on each country's welfare is proportional to their relative size. According to Clarida, Gali and Gertler (2002), the maximization of global utility (x.xx) over all periods of time can be solved for in a series of steps similar to the non-
coordinated case, and represented by

$$W_c^* = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t [ (1 - \gamma) (\pi_t^2 + \alpha \dot{y}_t^2) + \gamma (\pi_t^f + \alpha_f (\dot{y}_t^f)^2) - 2 \Phi \dot{y}_t \dot{y}_t^f ]$$  (2.06)

where $\Phi$ is a constant depending on the weights of the countries, price elasticities of demand, and the welfare benefit to consumption in the representative agents utility. The variable $\dot{y}$ carries a special meaning – the deviation of actual output from the level of output that would arise if prices were perfectly flexible worldwide. It is different from $\dot{y}_t$ in (2.07), which is defined as a similar gap, but with respect to output under domestically flexible prices, taking foreign prices as given. More care is taken to expound upon this difference in section (3.2).

This combination of non-cooperative objectives for the domestic and foreign country, and a coordinated objective provide numerical measures to compare policy. A monetary policy that raises (2.04) is better at maximizing domestic welfare, while a policy that raises (2.06) leads to greater global welfare in the sense of equation (2.05).

There have also been a number of other refinements to Obstfeld-Rogoff. Given the goal of this paper, the following are all framed in their ability to address the question of monetary policy coordination.

II.2.3 The Effects of Firm Pricing Behavior

One significant refinement has been the consideration of domestic firms that decide to price their exported goods in the currency of the foreign economies where the goods are sold. Betts and Devereux (2000) pioneer the work by assuming that a faction of firms can set different prices in home
and foreign markets. The most significant result to come from their work is that allowing firms to set prices in the local currency (LCP) allows for the possibility of exchange rate overshooting – a larger than necessary change in the exchange rate in response to a shock in the money supply. They argue that this process explains some of the exchange rate volatility found in empirical work. The drawback, however, is that the authors are not able to generate a model that persists abnormal exchange rate volatility beyond the time period were prices are assumed to be sticky. Essentially, even though Betts and Devereux are able to model exchange rate overshooting, it is sustained artificially through an exogenous price stickiness window.

Michaelis (2006) incorporates firm pricing behavior to directly access welfare benefits from monetary policy coordination. He zooms out to the country level by considering the percent of firms exhibiting local-currency pricing behavior out of the entire population of firms within a country. Furthermore, Michaelis allows this percentage (which can be regarded as susceptibility of an economy to exchange rate fluctuations, or 1 – the exchange rate pass through) to be different across the two countries.

He finds that under varying degrees of local-currency pricing, gains from international coordination in monetary policy can be vastly different. First, there is a critical share of domestic local-currency pricing (LCP) firms such that for any greater share, domestic monetary expansion raises domestic welfare. There is also a critical LCP share such that for any lower share, domestic monetary expansion raises foreign welfare. Thus, only for an intermediate range of the share does domestic monetary expansion expand both domestic and foreign economies (the two shares do not necessarily have to be equal). Home and foreign welfare maximized at equal fractions of firms pricing to market at 50%. Second, if the LCP firm shares of the two countries sum up to unity, the world Nash optimal monetary stance is identical for both countries and always supports the first-best allocation of each country.
Third and finally, Michaelis (2006) finds that there is always a benefit from monetary policy coordination, no matter what the share of LCP is in each country. This result is in stark contrast to numerous other works, including Corsetti and Pesenti (2005) and Devereux and Engel (2003), who argue against any gains from cooperation when the LCP share equals 0 or 1. The only exception to Michaelis’ rule is when the shares of local-currency firms in both countries sum up to unity (i.e. \( LCP_{\text{domestic}} + LCP_{\text{foreign}} = 1 \)). In this case coordination produces identical welfare to non-coordination. This result coincides with Betts and Devereux (2000).

\( \text{II.2.4. Non-Unitary Elasticity of Substitution Between Home and Foreign Goods} \)

Another important feature that almost all of the above cited papers assume (including Michaelis (2006)) is a unitary elasticity of substitution between home and foreign goods. Sutherland (2004) notes that this assumption rules out analysis of the expenditure switching effect of the exchange rate. This is a potentially significant factor in coordination analysis as it determines the effect of monetary policy on goods demands in different countries. Furthermore, Benigno and Benigno (2003) determine that unitary elasticity is the precise extreme case where the welfare gain from monetary policy coordination disappears.\(^5\) Taking Benigno and Benigno (2003) as given, the reason other authors have not found gains from coordination becomes readily apparent.

One potentially significant problem with allowing the elasticity of substitution of home and foreign goods to differ from 1 is that the resulting model becomes unsolvable. Luckily, second order approximation techniques exist to solve the problem. Such a technique produces solutions to stochastic models with no explicit solution which are accurate to the second-order, hence the name. This technique has been employed by Pappa (2002), Tchakarov (2002), Clarida, Gali and Gertler (2002) and

\(^5\) It should be noted that Michaelis’ assumption of unitary elasticity implicitly proves the exact opposite.
Sutherland (2004), to name a few.

The Sutherland (2004) model has the additional benefit of being simple enough to show explicitly the role of the expenditure switching effect in generating welfare. The uniqueness of Sutherland (2004) does not end there, though. It is also the only model thus far to compare different structures of international financial markets.

II.2.5 Varying the Structure of International Financial Markets

The crux of Sutherland (2004) lies in its analysis of the welfare effects of three different types of international financial markets: financial autarky, state-contingent asset trade taking place before central banks choose a monetary policy stance, and the same state-contingent asset trade taking place after central banks make their monetary policy decisions. In agreement with other literature, Sutherland finds welfare gains from monetary policy coordination when the elasticity of substitution of domestic and foreign goods is not unitary. He also finds that these gains are quantitatively very small when there is no international financial market, or financial autarky. On the other hand, when asset trading is allowed after central banks have made their monetary policy decisions, there is an additional spillover effect such that the non-coordinated choice of monetary policy adversely creates output volatility abroad. Asset trading before monetary policy makers make decisions produce the largest possible gains from coordination as agents can now fully insure themselves against any choice of monetary policy. Sutherland argues this is because non-coordinated central banks, seeing that agents are now fully ensured, create even more output volatility abroad, adversely affecting world welfare.

Sutherland finds that varying financial market structure can change the welfare benefits from coordination from producing as little as 0.2 % additional aggregate welfare (over the non-coordinated case) to producing over 100 % more welfare. The results in Sutherland (2004) are particularly
supportive of monetary policy coordination as the asset-trade-before-policy framework that produces
the biggest gains from coordination is also the structure, of the 3 considered, that most closely
describes international financial markets in the real world.⁶

II.2.6 Calvo Nominal Rigidity and Non-Logarithmic Consumption Preferences

A drawing point of the Obstfeld-Rogoff model is that it incorporates nominal rigidity, which is
theorized as the reason nominal economic shocks can have real economic impacts. This channel is, of
course, essential to monetary policy coordination. The method of simply assuming prices are held
constant for two periods, on the other hand, is not as enticing. Walsh (2003, pg. 222-23) shows that
while Obstfeld-Rogoff can emulate the sort of sluggish behavior of prices that is commonly observed,
it misses out on incorporating the same sort of sluggish behavior in inflation. Setting prices fixed for
only two periods takes care of stickiness in the level effects of prices, but does not explain the
additional stickiness in the rate of change of prices.

Clarida Gali and Gertler (2002) adapt an approach first proposed in Calvo (1983) to model
nominal rigidity. The approach assumes that each period, only a fraction of firms get the chance to set
prices optimally in response to economic conditions. The rest of the firms simply keep the same prices
they had in the previous period. As the chance to re-optimize prices is assumed to follow an Poisson
process, the approach has the advantage that some firms will not be able to adjust prices for a long
time, while others will reset fairly frequently. This not only brings a micro-economic foundation to why
aggregate prices are sluggish to fully adjust, but has the additional benefit of showing a sluggish
response in inflation exactly because some firm prices do not change for a considerable time.

The above definition of nominal rigidity, along with an assumption of non-logarithmic

⁶ This is due to the fact that equity markets can implicitly provide insurance against nominal interest rate risk.
consumption preferences, is shown by Clarida Gali and Gertler (2002) to be sufficient conditions to have gains from coordinating monetary policy. In light of the previously described extensions, Clarida, Gali and Gerter (2002) assumes no asset based financial market structure, and complete exchange rate pass-through by purchasing power parity, the exact conditions under which gains from coordination did not appear in other papers. This contradiction is resolved in the fact that the other studies assume logarithmic consumption preferences, in which case Clarida Gali and Gertler (2002) find no gains to coordination as well.

Clarida, Gali and Gertler (2002) has the additional benefit of explicitly deriving the monetary policy rules a central bank can follow in order to achieve a coordinated policy. It seems Branson's advice did not fall on deaf ears. Two decades after his publication, the choice to coordinate is squarely in the hands of central banks.  

II. 3 From Theory to Numerical Analysis

As the above discussion may illustrate, the recent progression of macroeconomic policy analysis and expansion of the general equilibrium modeling literature, especially in the field of micro-founded macro models, has created a plethora of macroeconomic models of all shapes and sizes. With these new models there has come a need for a unified and rigorous approach to numerical analysis. Dynare is one software package designed to fulfill this need, and is employed for the numerical analysis of this paper. It is publicly available software designed to

“solve non-linear models with forward looking variables... Although it can be put to other use, Dynare has been built in order to study the transitory dynamics of non-linear models with consistent expectations. “(Dynare, 2008)

7 Incidentally, Clarida, Gali and Gertler (2002) was also partially funded by the NSF.
In this section, I will describe the algorithms Dynare uses to solve both deterministic and stochastic models.

II.3.1 Deterministic Solutions

The approach to solving deterministic models in Dynare uses a Newton-Raphson algorithm detailed in Juillard (1996) and introduced in Laffargue (1990). I will first define the general algorithm before showing how a standard macroeconomic model can be adapted to find a solution. Newton-Raphson proceeds iteratively. At each step, the solution to \( F(\bar{Y}) = 0 \) is calculated, where \( F(\bar{Y}) \) is a system of (perhaps non-linear) equations, taking the matrix \( \bar{Y} \) as inputs, and \( \bar{0} \), the zero matrix. After solving the system, \( F(.) \), with an initial guess, successive iterations are obtained by altering the previous estimate by a function of the Jacobian matrix of \( F \). The algorithm stops when two general cases are satisfied: (1) the solution of a guess is sufficiently close to 0, and (2) successive guesses are converging. In terms of the previously defined system, Newton-Raphson evaluates:

\[
F(Y^i) \text{ where } Y^i = Y^{i-1} - [\frac{\partial F}{\partial Y}(Y^{i-1})]^{-1} F(Y^{i-1})
\] (2.07)

and \( \frac{\partial F}{\partial Y} \) is the Jacobian matrix \( \left[ \frac{\partial F}{\partial y_i} \times \frac{\partial F}{\partial y_j} \right] \) for \( i, j < n \) the number of distinct variables in the system, at each iteration, stopping when \( F(Y^i) \) is close to 0 and \( |Y^{i-1} - Y^i| < \epsilon \) where \( \epsilon \) is a predetermined value. In general, Judd notes that we have to very careful to pick an appropriate initial guess, as the convergence theorem for Newton methods, Theorem 5.5.1 in Judd (1998, pg 168), holds only for initial values in a neighborhood of the actual solution to the system.
In the scope of this paper, since all the models are already linearized, the Newton-Rhapson algorithm simplifies to a single step, finding the solution to \( F(\bar{Y}) = A\bar{Y} = 0 \). A linear method, such as Gauss-Siedel\(^8\), is then used to update an initial guess for \( \bar{Y} \) until a suitable solution is found.

The Juillard approach can be applied to a standard macroeconomic model by defining \( f_n(y_{t+k}, \ldots, y_t, \ldots, y_{t-l}, z_t, \theta) = 0 \), the solution to a system of equations that govern an economy based on endogenous variables, \( y_t \), expressed in both leads and lags, exogenous variables, \( z_t \), and parameters \( \theta \) that do not depend on time. Since in a deterministic setting the exogenous variables \( z_t \) and parameters, \( \theta \) are given, we can write one system of equations to represent all states of the economy over a time horizon \([0, \ldots, T]\) as:

\[
F(Y) = \begin{bmatrix}
    f_{1-k}(y_{1-k}) \\
    \vdots \\
    f_{0}(y_{0}) \\
    f_{1}(y_{1-k}, \ldots, y_{1+l}) \\
    \vdots \\
    f_{T}(y_{T-k}, \ldots, y_{T+l}) \\
    f_{T+1}(y_{T+1}) \\
    \vdots \\
    f_{T+l}(y_{T+l}) \\
\end{bmatrix} = 0 \quad \text{where} \quad Y = \begin{bmatrix}
    y_{1-k} \\
    \vdots \\
    y_{0} \\
    y_{1} \\
    \vdots \\
    y_{T} \\
    y_{T+1} \\
    \vdots \\
    y_{T+l} \\
\end{bmatrix} \tag{2.08}
\]

and \( y_0, y_T \) represent initial and terminal conditions respectively. Such a model exactly describes a system of equations that Newton-Rhapson can solve, giving the value of endogenous variables over all periods of time for a given deterministic shock, values of \( u_t, \forall t \in [0, \ldots, T] \). Boucekkine (1995) reviews procedures to find suitable solution horizons, \( T \), in order for solutions to converge to terminal conditions.

---

\(^8\) See Judd (1998, pg. 72) for details.
II.3.2 Stochastic Solutions

While solutions to deterministic models are computationally straightforward and numerical methods can reasonably guarantee a solution, solving stochastic models is more involved. The complication is that we can no longer assume that the exogenous shock process, \( z_t \), is predetermined. Any solution will be a series of functions representing endogenous variables at time \( t \) as functions of predetermined variables and the stochastic process. It is not clear that an analytical solution to such a problem even exists.

Earlier methods, as described in Taylor (1993, Ch 1), involved solving the deterministic solutions for numerous random draws of \( z_t \) from an assumed distribution. Averaging across these draws gave a numerical approximation for the moments, and thus stochastic characteristics, of endogenous variables. Although such methods do allow for policy simulation, and have the benefit of being direct extensions of deterministic methods, a need existed for sharp, analytical results. First, an analytical solution would have the benefit of much shorter computation time after the initial formula was found. And, second, an existence theorem would have the benefit of allowing researchers to look at the necessary economic assumptions for a viable solution. This has become especially important given the prevalence of micro-founded macroeconomic models already described.

Klein (2000) contributes a recent method for analytically solving multivariate linear rational expectations models. The general problem can be defined as follows. Let \((\Omega, F, P)\) be a given probability space and \( \bar{F} = \{ F_t ; t = 0, 1, \ldots \} \) be a filtration of \( F \). If \( z = \{ z_t ; t = 0, 1, \ldots \} \) is an exogenous \( \bar{F} \) adapted stochastic process, \( A \) and \( B \) are \( n \times n \) matrices and \( C \) an \( n \times n_z \) matrix where \( n_z \) is the dimensionality of \( z \), then the following conditions (i) – (v) are sufficient to guarantee existence of a solution to the following system (for an \( n \)-dimentional process \( x \):}
\[ AE[x_{t+1}|F_t] = Bx_t + Cz_t, \quad t = 0, 1, \ldots \quad (2.09) \]

(i) \( z \) is stable and adapted to the filtration \( \bar{F} \).

(ii) some of the variables in the vector \( x_t \) are “backward looking.” Where backward looking is defined to hold if the prediction error, \( x_{(i,t+1)} - E[x_{(i,t+1)}|\bar{F}_t] \), is an exogenous martingale process and \( x_{(i,0)} \in \bar{F}_0 \) is given, for \( i \) variables in \( x_t \).

(iii) there exists \( z \in \mathbb{C} \) such that \( |Az - B| \neq 0 \)

(iv) there is no \( z \in \mathbb{C} \) with \( |z| = 1 \) and \( |Az - B| = 0 \)

(v) there are as many “backward looking,” or predetermined variables as there are stable generalized eigenvalues of \( P(z) = Az - B \), where stability is defined by modulus 1.

Before giving a general overview of the method used in Klein (2000), I will briefly go over how each of the five assumptions are guaranteed in a general SDGE model. The first assumption is almost always guaranteed by assuming the exogenous shock follows an AR(1) process with mean zero and some covariance matrix \( \phi \). A “backward looking” variable is a more generalized form of variables that do not enter the system as future expectation, or are strictly predetermined. Conditions (iii) and (iv) guarantee that the complex generalized Shur form of the matrices A and B exist,\(^9\) and that

\[ P(z) = Az - B \text{ has a set of eigenvalues that are defined by } \lambda(A, B) = \left\{ \frac{t_i}{s_i} : s_i \neq 0 \text{ or } t_i \neq 0 \forall i \right\}. \]

Condition (v) can be recognized as identical to the standard Blancard-Kahn conditions, detailed in Blancard and Kahn (1980). Note also that the above conditions and results still hold if A,B are real matrices. Hence, the conditions required for a solution by the Klein (2000) method are completely

\(^9\) For reference, the complex generalized Shur form is given by \( QAZ = S, QBZ = T \) both upper triangular and \( Q, Z \) are \( n \times n \) unitary matrices.
covered by assuming that the Blancard-Kahn conditions hold, the system has no unitary roots and the stochastic shock follows an AR process.

The method of solution follows by first defining auxiliary variables \( y_t = Z^H x_t \) and partitioning by \( y_t = \begin{bmatrix} s_t \\ u_t \end{bmatrix} \) where \( s_t \) and \( u_t \) correspond to stable and unstable generalized eigenvalues of the system \( (x.x) \) with \( y_t \) in place of \( x_t \). The next step is triangulating the system, which is given by the Shur decomposition, pre-multiplying and partitioning into the following form:

\[
\begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix} E \begin{bmatrix} s_{t+1} \\ u_{t+1} \end{bmatrix} F_t = \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} s_t \\ u_t \end{bmatrix} + \begin{bmatrix} Q_I \\ Q \end{bmatrix} C z_t \tag{2.10}
\]

since \( S \) and \( T \) are upper triangular by the assumed conditions. Then the set of variables pertaining to unstable variables, \( u_t \), are solved first, using the fact that \( S_{22} \) and \( T_{22} \) are also upper triangular and the stochastic properties of \( z_t \). The resulting equation for unstable variables is of the form

\( u_t = M z_t \). The stable variables are then solved for using the facts that the unstable variables are now known and our assumption of prediction errors following an exogenous martingale process. Having found formulas for the stable and unstable parts of \( y_t \), the solution, \( x_t \), is obtained by inverting the original transformation.

The method used in Dynare, and thus in the following analysis, is technically an extension of Klein's method to non-linear models and is reviewed in Collard and Juillard (2001). Juillard shows that the first order Taylor approximated solution to a non-linear stochastic system is given by

\[
x_t = \bar{x} + g_x \hat{x}_{t-1} + g_z z_t \tag{2.11}
\]
where \( x_t = g(x_{t-1}, z_t) \) is the actual solution to the non-linear system, for some function
\[
g : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n, \quad \text{and} \quad \bar{x} \quad \text{a deterministic steady state.}^{10}\]

The resulting variance-covariance matrix, \( \Sigma_x \), is then given by

\[
\Sigma_x = g_x \Sigma_x g_x^T + \sigma^2 (g_z \Sigma_z g_z^T) \quad (2.12)
\]

where \( T \) indicates a transpose. Equation (2.12) can be recognized as a Lyapunov equation and thus solved by any one of numerous optimization techniques, for instance, those covered in Judd (1998, Ch. 4). The above discussion illustrates the computation of theoretical variances that are presented in section (x.x)

The next section will describe the two models used to test gains from coordination.

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10 The analogous linear stochastic system would be given by \( x_t = Ex_{t-1} + Fz_t \), \( E,F \) matrices.
III. COMPARING CCG AND T85

The two models I will be comparing are a rational expectations model taken from the first chapter of Taylor 1993 and the micro founded, general equilibrium model taken from Clarida, Gali and Gertler 2002. I will hereafter refer to them as T85 and CCG, respectively. Both models are log-linearized around steady states, so any variable, with the exception of interest and inflation rates, are measured as logarithms, and all variables can be thought of as the deviation from a log run trend. Throughout the analysis I will assume that both home and foreign countries have identical equations.

Although based on two different approaches, both CCG and T85 have most of their differences grounded in their assumptions, rather than any general difference arising from construction. For instance, both CCG and T85 have similar output equations (see tables 3.1 & 3.2). Output this period depends negatively on the gap between nominal interest rates and expected inflation. T85 at first seems more open to fluctuations abroad, as it incorporates foreign output directly, but output under CCG is also exposed to foreign output through the definition of the long run real interest rate under flexible prices. Prices seem to enter the output equations through two different channels – expected inflation for CCG and current price level for T85 – but I will explain later that this is largely due to the definitions of price level and inflation in either model.

CCG diverges from T85 in output, by their definition of output under both domestically and globally flexible prices. Even with perfectly flexible prices and wages, it is assumed that output fluctuations may exist, mainly from international linkages, and residual effects of previous wage shocks. As will be later evident, a central point to coordination analysis is that CCG distinguishes between the event when a domestic country assumes flexible prices, but takes foreign prices as given,

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11 I will not review the specific construction of CCG from optimizing firms and households. Instead I only cite derivations where they serve to illustrate a comparison to T85. A full construction of CCG from basic assumptions can be found in Clarida, Gali and Gertler (2002)
and when prices are flexible globally. Furthermore, CCG defines output, $y_t$, as the gap in output between aggregate output, $\bar{y}_t$, and the level of output under domestically flexible prices only, $\bar{y}$, (CCG1.F). There is also an analogous output gap, $\hat{y}_t$, that measures the difference between aggregate output and the level of output under globally flexible prices. The output gap under globally flexible prices is defined by its relation to the domestically flexible prices output gap, $y_t$, as can be seen from (CCG1.H).

T85 effectively abstracts from these definitions by assuming output is measured in real terms. Hence all long run terms under flexible prices are identically 0. Incidentally, this is also why there is no long run real interest rate term in (T851.A).
Country 1 Equations

\[ y_t = E_t[y_{t+1}] - \sigma_0^{-1}(i_t - E_t[\pi_{t+1}] - \bar{\rho}) \]  

(CCG1.A)

\[ \bar{\rho} = \sigma_0(E_t[\bar{y}_{t+1}] - \bar{y}_t) + \kappa_0(E_t[\bar{y}_t^f] - \bar{y}_t^f) \]  

(CCG1.B)

\[ \pi_t = \beta E_t[\pi_{t+1}] + \lambda y_t + u_t \]  

(CCG1.C)

\[ mc_t = u_t + \kappa \bar{y}_t + \kappa_0 \bar{y}_t^f \]  

(CCG1.C2)

\[ ur_t = \rho u_{t-1} + \epsilon_t \]  

(CCG1.E)

\[ \bar{y}_t = y_t + \bar{y}_t \]  

(CCG1.F)

\[ \bar{y}_t = -\frac{\kappa_0}{\kappa} \bar{y}_t^f \]  

(CCG1.G)

\[ \hat{y}_t = y - \frac{\kappa_0}{\kappa} \bar{y}_t^f \]  

(CCG1.H)

Country 2 Equations

Identical to country 1 with coefficients changed to foreign values, see Table (x.x)  

(CCG2)

Definition of Variables

\[ y_t = \text{Output gap, difference between output and domestic flexible output prices} \]

\[ \bar{y}_t = \text{Output under domestically flexible prices (taking foreign prices as given)} \]

\[ \hat{y}_t = \text{Aggregate output} \]

\[ \hat{y}_t = \text{Output gap under globally flexible prices} \]

\[ \pi_t = \text{Inflation rate} \]

\[ mc_t = \text{Marginal cost} \]

\[ \bar{\rho} = \text{Long run real interest rate under domestically flexible prices} \]

\[ ur_t = \text{Exogenous cost shock} \]

\[ f = \text{(as a superscript) denotes a foreign variable / coefficients} \]

Table 3.1 Equations and Variable Definitions of CCG

---

12 All variables except inflation, nominal and real interest rates are log deviations from steady state. Inflation is denoted as deviation from steady state.
Country 1 Equations

\[ y_t = -\Delta r_t + f (e_t + p_t^f - p_t) + g_t^f \]  
(T851.A)

\[ x_t = \frac{\delta^{T85}}{3} \sum_{i=0}^{2} w_{t+i} + \left(1 - \frac{\delta^{T85}}{3}\right) \sum_{i=0}^{2} p_{t+i} + \frac{y^{T85}}{3} \sum_{i=0}^{2} y_{t+i} + u_t \]  
(T851.B)

\[ w_t = \frac{1}{3} \sum_{i=0}^{2} x_{t-i} \]  
(T851.C)

\[ p_t = \theta^{T85} w_t + (1 - \theta^{T85})(e_t + p_t^F) \]  
(T851.D)

\[ \pi_t = p_t - p_{t-1} \]  
(T851.E)

\[ r_t = i_t - E [\pi_{t+1}] \]  
(T851.F)

\[ u_t = \rho u_{t-1} + \epsilon_t \]  
(T851.G)

Country 2 Equations

*Identical to country 1 with coefficients changed to foreign values, see Appendix (x.x)*  
(T852)

**Definition of Variables**

- \( y_t \) = Real GNP
- \( p_t \) = Price level
- \( i_t \) = Nominal interest rate
- \( r_t \) = Real interest rate
- \( \pi_t \) = Inflation rate
- \( w_t \) = Nominal wage
- \( x_t \) = Contract wage
- \( e_t \) = Exchange rate; country 1 price of country 2 currency
- \( f \) = (as a superscript) denotes a foreign variable / coefficients

---

13 All variables except inflation, nominal and real interest rates are log deviations from steady state. Inflation is denoted as deviation from steady state.
<table>
<thead>
<tr>
<th>Coefficient Name</th>
<th>Symbol</th>
<th>Equation</th>
<th>Range</th>
<th>Baseline Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree of Consumption Preference</td>
<td>( \sigma )</td>
<td>N/A</td>
<td>([0,1])</td>
<td>0.2</td>
</tr>
<tr>
<td>Degree of Leisure Dis-utility</td>
<td>( \phi )</td>
<td>N/A</td>
<td>((-1,\infty))</td>
<td>1</td>
</tr>
<tr>
<td>Foreign Country Mass (relative to home)</td>
<td>( \gamma )</td>
<td>N/A</td>
<td>((0,1))</td>
<td>0.5</td>
</tr>
<tr>
<td>Future Discount Term</td>
<td>( \beta )</td>
<td>N/A</td>
<td>([0,1])</td>
<td>0.99</td>
</tr>
<tr>
<td>Probability that a firm keeps its price fixed</td>
<td>( \theta )</td>
<td>N/A</td>
<td>((0,1))</td>
<td>0.67</td>
</tr>
<tr>
<td>Price Elasticity of Demand of Intermediate Goods</td>
<td>( \xi )</td>
<td>N/A</td>
<td>((0,\infty))</td>
<td>1</td>
</tr>
<tr>
<td>Shock Persistence</td>
<td>( \rho )</td>
<td>N/A</td>
<td>((0,1))</td>
<td>0.95</td>
</tr>
<tr>
<td>Elasticity of Marginal Cost (with respect to domestic output)</td>
<td>( \kappa )</td>
<td>( \sigma(1-\gamma)+\gamma+\phi )</td>
<td>((-1,\infty))</td>
<td>1.6</td>
</tr>
<tr>
<td>Elasticity of Marginal Cost (with respect to foreign output)</td>
<td>( \kappa_0 )</td>
<td>( \sigma \gamma - \gamma )</td>
<td>((-1,0))</td>
<td>-0.4</td>
</tr>
<tr>
<td>Elasticity of Output (with respect to real interest rate)</td>
<td>( \sigma_0 )</td>
<td>( \sigma - \kappa_0 )</td>
<td>([0,1])</td>
<td>0.6</td>
</tr>
<tr>
<td>Elasticity of Inflation (with respect to output)</td>
<td>( \lambda )</td>
<td>( \delta \kappa )</td>
<td>((-\infty,\infty))</td>
<td>0.27</td>
</tr>
<tr>
<td>Elasticity of Marginal Cost (with respect to domestic output) – Country 2</td>
<td>( \kappa^f )</td>
<td>( \sigma \gamma(1-\gamma)+\phi )</td>
<td>((-1,\infty))</td>
<td>1.6</td>
</tr>
<tr>
<td>Elasticity of Marginal Cost (with respect to foreign output) – Country 2</td>
<td>( \kappa_0^f )</td>
<td>( \sigma(1-\gamma) )</td>
<td>((-1,0))</td>
<td>-0.4</td>
</tr>
<tr>
<td>Elasticity of Output (with respect to real interest rate) – Country 2</td>
<td>( \sigma_0^f )</td>
<td>( \sigma - \kappa_0^f )</td>
<td>([0,1])</td>
<td>0.6</td>
</tr>
<tr>
<td>Elasticity of Inflation (with respect to output) – Country 2</td>
<td>( \lambda^f )</td>
<td>( \delta \kappa^f )</td>
<td>((-\infty,\infty))</td>
<td>0.27</td>
</tr>
<tr>
<td>Elasticity of Inflation (with respect to marginal cost)</td>
<td>( \delta )</td>
<td>( \frac{(1-\theta)(1-\beta \theta)}{\theta} )</td>
<td>((0,\infty))</td>
<td>0.17</td>
</tr>
<tr>
<td>Elasticity of Inflation under optimal, discretionary, non-coordinated policy (with respect to domestic cost shock)</td>
<td>( \psi )</td>
<td>( [1-\beta \rho+\xi \lambda]^{-1} )</td>
<td>((-\infty,\infty))</td>
<td>3.08</td>
</tr>
<tr>
<td>Coefficient in expression of optimal coordinated policy in terms of cost shocks (x.xx)</td>
<td>( \bar{\psi} )</td>
<td>( [1-\beta \rho+\xi (\lambda-\delta \kappa_0)]^{-1} )</td>
<td>((-\infty,\infty))</td>
<td>2.56</td>
</tr>
<tr>
<td>Weight on output deviations in loss function</td>
<td>( \alpha )</td>
<td>( \lambda/\xi )</td>
<td>((-\infty,\infty))</td>
<td>0.27</td>
</tr>
</tbody>
</table>
Weight on output deviations in loss function – Country 2  
\[ \alpha' \quad \frac{\lambda_f / \xi}{\xi} \quad (\ -\infty \ , \ \infty \ ) \quad 0.27 \]

Weight on coordinated output in coordinated loss function  
\[ \Phi \quad \frac{\delta (1-\sigma) \gamma (1-\gamma)}{\xi} \quad (0, \ \infty) \quad 0.033 \]

Coefficient on expected inflation in discretionary policy rule (x.xx)  
\[ \theta_1 \quad \left( 1 + \frac{\xi \sigma_0 (1-\rho)}{\rho} \right) \quad (1, \ \infty) \quad 1.03 \]

Coefficient on foreign expected inflation in coordinated policy rule (x.xx)  
\[ \theta_2 \quad \frac{\kappa_0}{\kappa} (\theta_1 - 1) \quad (\ -\infty \ , \ \infty \ ) \quad -0.008 \]

Coefficient on expected inflation in simple policy rule (x.xx)  
\[ \theta_{rule} \quad 1 + \frac{\sigma_0 \xi}{(1-\beta \rho)} \left( 1 - \rho \right) \quad (1, \ \infty) \quad 1.53 \]

<table>
<thead>
<tr>
<th>Coefficient Values for Simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta^{T85} = 0.5 )</td>
</tr>
<tr>
<td>( \gamma^{T85} = 1.0 )</td>
</tr>
<tr>
<td>( \theta^{T85} = [0,1] )</td>
</tr>
<tr>
<td>( d = 1.2 )</td>
</tr>
<tr>
<td>( f = 0.1 )</td>
</tr>
<tr>
<td>( g = 0.1 )</td>
</tr>
<tr>
<td>( b = 4.0 )</td>
</tr>
<tr>
<td>( a = 1.0 )</td>
</tr>
</tbody>
</table>

Table 3.3 Coefficient Definitions and Values for CCG
III. 1 Defining Price Stickiness

Perhaps the largest constructive difference between CCG and T85 lies in how the respective models define nominal rigidity in prices. I will summarize the differences in this section and comment on the resulting affects on the linearized equations and equilibrium effects. Many of the subsequent points are extensions from the analysis found in Walsh (2003).

Price rigidity is incorporated into T85 through staggered wage contracting. Essentially, it is assumed that households do not work under the pretext that their wage can change continuously at the whim of the firm. Instead, every worker is employed under contract at a fixed wage for some period of time, and different workers renew their contracts at different dates. Unanticipated nominal shocks can now have an affect on real variables since any firm with workers in the middle of a contract will not be able to instantaneously adjust wages to respond to the shock. Firms (workers) may have to adjust employment (consumption), which in turn affects on output.

In the canonical T85 model, all contracts are assumed to be 3 periods in length\(^4\) and an equal percent of the population renegotiate their contacts at any given time period. Hence the contact wage today is defined to be a weighted average of the annual wage, price, and output today and expected values of the previous three variables over the next two periods.

\[
x_t = \frac{\delta}{3} \sum_{i=0}^{2} w_{t+i} + \frac{1-\delta}{3} \sum_{i=0}^{2} p_{t+i} + \frac{\nu}{3} \sum_{i=0}^{2} y_{t+i} + u_t \tag{3.01}
\]

The average wage in the economy is then just the average of contract wages over the past 3 periods.

\(^4\) Equivalently, resetting very 4\(^{th}\) period, which motivates the viewpoint of each period representing ¼ year.
To illustrate the implications of staggered contract wages, assume that prices are a constant, normalized markup over wages $p_t = w_t$ and inflation is defined as $\pi_t = p_t - p_{t-1}$. One can then show that prices and inflation in T85 evolve according to the following two equations:\(^{15}\)

$$p_t = \frac{1}{6} (2p_{t-1} + p_{t-2} + 2E_t p_{t+1} + E_t p_{t+2} + \eta_t^p + \zeta(y_t))$$  \hspace{1cm} (3.03)

$$\pi_t = E_t \pi_{t+1} + \frac{1}{3} (E_t \pi_{t+2} - \pi_{t-1}) + \frac{1}{3} (\eta_t^p + \zeta(y_t))$$  \hspace{1cm} (3.04)

where $\eta_t^p = (E_t p_t - p_t) + (E_{t-1} p_{t+1} - E_t p_{t+1}) + (E_{t-2} p_{t-1} - p_{t-1}) + (E_{t-2} p_{t-1} - p_t)$ is an expectational error term and $\zeta(y_t) = y_{t-2} + 2y_{t-2} + 3y_t + 2E_t y_{t+1} + E_t y_{t+2} + \eta_t^y$. Staggered wage contracts of 3 periods in length lead to prices depending on previous levels and expected future levels for two periods in either direction. The intuition is straightforward as at any point in time, a third of contracts will reflect wages set optimally two periods in the past and any contracts determined today will still be in use two periods into the future.

What is perhaps more surprising is that (3.04) shows inertia in the inflation rate. This implies that inflation cannot be reduced without a cost to output. For instance, consider a policy that shifts inflation from an equilibrium constant rate to a lower rate and maintains the new level. When the policy is announced, the expectation gap $\left(E_t \pi_{t+2} - \pi_{t-1}\right)$ will be non-zero, implying that either current or expected output must change. In contrast, Walsh shows that no such inertia is present when contracts of

---

$^{15}$ See appendix (x.x)
only two periods are considered.\textsuperscript{16}

In contrast, CCG assumes a Calvo model of staggered price adjustment.\textsuperscript{17} Under Calvo, firms face stickiness in pricing their goods, as opposed to setting wages. Only a fraction of firms are able to update prices each period, and the opportunity for an individual firm to adjust prices arrives according to an exogenous Poisson process. The probability of adjusting is set exogenously to be \(1 - \omega\). Through their derivations, and the additional assumption that wages are perfectly flexible except for exogenous variation in the markup,\textsuperscript{18} given by \(\epsilon_t\), Clarida Gali and Gertler show that inflation evolves in CCG according to:

\[
\pi_t = \beta E_t[\pi_{t+1}] + \lambda y_t + u_t
\]  

(3.05)

Comparing equations (3.04) and (3.05) we notice that, overall, inflation in both models depends on expected inflation, current output, and an error term. There are also a variety of differences. First, inflation in T85 depends on both lags and leads of both inflation and output, while in CCG, inflation today is defined only by expected inflation tomorrow and output today. As noted above, (3.05) implies that inflation can be lowered without a cost to output in CCG, while not in T85. An interesting side note is that changing our assumption about contract lengths in T85 to 2 quarter contracts equally distributed among the population will yield an inflation equation much closer to (3.05) (see footnote #3). This will be important when T85 is eventually used to check the robustness of policy results in CCG.

A second difference is that CCG takes into account discounting of future prices and income through the coefficient \(\beta\), while T85 does not. It turns out that it is not difficult to incorporate future discounting into T85 by analyzing how a representative agent may view the equations in the context of

\textsuperscript{16} In fact, inflation evolves according to: \(\pi_t = E_t[\pi_{t+1}] + 2(y_t + y_{t-1}) + \eta_t\), see Appendix (x.x)

\textsuperscript{17} First introduced in Calvo (1983)

\textsuperscript{18} Clarida, Gali and Gertler give one possible definition of this exogenous markup as shifts in workers' market power.
an inter temporal budget constraint.\textsuperscript{19} It also does not affect the resulting analysis in this paper, so I do not consider the point further.

The third and final difference is more subtle, but worth noting. By assuming wage contracts of 3 periods in length, T85 assumes that in any given period 1/3 of the wages will be able to adjust. Furthermore, this 1/3 will be uniquely different for every set of 3 periods, so that all wages will have adjusted by the 4\textsuperscript{th} period. On the other hand, CCG assumes that a random percent of firms will be able to adjust prices every period. For instance, letting \((1-\theta)=1/3\) , or that 1/3 of firms will be able to adjust prices every period implies that a \((2/3)^4\approx 0.198\) proportion of firms will still not have had a chance to adjust prices by period 4. We can lessen the difference in persistence between the two models by assuming shorter contract lengths (equivalently a lower \(\omega\)).

In CCG, the coefficient, \(\lambda\) , controls the impact of output today on inflation in the Phillips style equation. As can be expected, setting the probability that firms can not adjust prices to 1 fixes \(\lambda\) at 0. In this case inflation is defined completely by expected inflation tomorrow and the exogenous variation in wage markup discussed in the previous paragraph. On the other hand, approaching a scenario where firms are able to adjust prices every period causes \(\lambda\) to increase and the output effect to dominate.

Throughout the preceding analysis, we assumed that prices in T85 evolved according to a constant markup over wages, \(p_t = w_t\). In contrast, the standard T85 model assumes a relaxed purchasing power parity relation, that changes the above prices equation into one of the form,

\[ p_t = \theta w_t + (1-\theta)(e_t + p^F_t) \]

where \(0<\theta<1\) denote a neither a closure of open economy effects on prices ( \(\theta=1\) ), nor strict purchasing power parity ( \(\theta=0\) ). As shown in Appendix (A.3), assuming relaxed PPP in the uniformly distributed, 2 contract environment gives the resulting analogues for price level and inflation to the above analysis,

\textsuperscript{19} Again, Walsh (2003, pp) shows this for the case of 2 period contract lengths
\[
p_t = \frac{1}{2\phi} \left( \phi E_t \pi_{t+1} + \phi \pi_{t-1} + \phi \pi_t \right) + (1-\theta)(4\pi_t' - \delta(\pi_{t-1}' + 2\pi_t' + E_t\pi_{t+1}' + \pi_t')) \\
\pi_t = E_t\pi_{t+1} + \left( 1-\theta \right) \left( \frac{\delta}{\phi} \left| E_t\pi_{t+1}' - \pi_t' \right| \right) + \theta \left( \frac{1}{\phi} \left| 4\delta - 4 \right| (w_t - p_t) + y \zeta^1(y_t) \right) + \eta'' \quad (3.07)
\]

where \( \phi = \delta(1-\theta) + \theta \). As can be expected, introducing open economy effects changes the resulting price level equation to depend on foreign prices as well as domestic prices. Essentially, the foreign terms in (3.06) measure the difference between the current foreign price level and a weighted dispersion of foreign prices across all contracts currently active. This is analogous to measuring the difference between what the foreign price level would be under closed prices and what it actually is.\(^{20}\)

Another interesting point is that the other domestic terms in the price level are now weighted against foreign effects according to the degree of openness, \( \theta \).

The resulting inflation equation, (3.07), offers the same results. As in the closed prices case, inflation today depends on the expectation of inflation tomorrow and domestic output dispersion across active contracts. But now, foreign inflation has a direct effect on domestic inflation. As for the price level, domestic inflation does not depend on the level of foreign inflation, but on the expected change in inflation. We also see additional domestic terms in the relaxed PPP inflation equation. Since we no longer have the relation that wages are a constant markup over prices, the difference term \( w_t - p_t \) does not cancel out. An explicit measure of the time varying wage markup now affects domestic inflation.

In conclusion, although T85 and CCG subscribe to two different schools of describing nominal rigidities, it is possible, after a few basic assumptions, to show they have similar effects on how

\(^{20}\) To see this, compare equations (x.xx) and (x.xx).
inflation evolves. The additional difference of CCG being micro-founded and T85 constructed from macroeconomic principles gives an impressive array of aspects to analyze. Increasing contract lengths in T85 can be considered economically similar to raising $\omega$, the probability firms are be able to adjust prices. Though, as shown above, increasing contract lengths above 2 periods introduces inflation persistence in T85 while more rigidity in CCG lessens the impact of current output on inflation. Also, to produce similar inflation equations, we had to assume prices were closed to the rest of the world in T85. This gives evidence that, the assumption of PPP in CCG is ultimately not expressed. In light of coordinating monetary policy, shutting off such an effect may have significant consequences to the eventual gains (or lack thereof) from coordination.

II. 2 Open Economy Effects

Open economy effects enter CCG through a few main channels. The first channel, though marginal cost, is considered by Clarida, Gali and Gertler to be preeminent. Equation (CCG1.C3) shows that domestic marginal cost evolves according to the domestic cost shock, output and foreign output. The effect of the cost shock, which can be perceived as an increase in worker's wages\(^{21}\), unambiguously increases marginal cost. The effects of domestic and foreign output, on the other hand, are determined by the coefficients $\kappa$ and $\kappa_0$, both of which depend on the foreign country mass, relative to the domestic country, $\gamma$.

The effect of foreign output on domestic marginal cost is governed by, $\kappa_0$, which by Table (3.1) depends on $\sigma \gamma$ and $-\gamma$. This can be perceived as the presence of two conflicting effects of foreign output on marginal cost. First, a rise in foreign output increases the supply of imports to the domestic country, lowering the price of imports and raising the domestic price index relative to abroad.

\(^{21}\) This will be explained in full detail in a later section.
– the terms of trade effect. Since workers' wages are denominated in domestic prices, nominal wages fall and labor supply increases, which decreases firms' marginal costs. This effect is captured by the term \(-\gamma\). On the other hand, the increase in foreign output also raises domestic consumption of foreign goods. Increased consumption raises the marginal rate of substitution between consumption and leisure, leading to less incentive to work, raising marginal cost. As can be expected, the latter effect is captured by \(\sigma\gamma\), and dubbed the risk sharing effect. Which effect dominates depends on the values of coefficients assumed. This paper considers a baseline version of CCG where \(\sigma = 0.2\) and \(\gamma = 0.5\), hence \(\kappa = -0.4\) and the terms of trade effect wins out.

Similarly, the coefficient on domestic output, \(\kappa\), depends on \(\gamma\). Increasing domestic output increases marginal cost due to the inverse of the terms of trade effect described above. Clarida, Gali and Gertler note that the effect is dampened due to global risk sharing. Though, while open economy effects can work to either increase or decrease the elasticity of marginal cost to domestic output, the effect will be unambiguously positive. For symmetry, in the baseline CCG model, we have \(\kappa = 1.6\), hence an increase in output has more than a 1-for-1 effect on marginal cost.

When equations (CCG1.C2) and (CCG1.C3) are combined, they produce the Phillips-style equation, (CCG1.C). It is interesting to note that this causes the foreign output term to drop out, and \(\gamma\) to only appear in the coefficient on domestic output, \(\lambda\). As stated earlier, the value of \(\lambda\) is dominated by \(\theta\), a proxy for the stickiness of firms setting prices. Varying \(\gamma\) between 0 and 1 produces no significant variation in \(\lambda\). For the purposes of numerical analysis, (CCG1.C2) and (CCG1.C3) are kept separate to be able to distinguish open economy effects on marginal cost.

The second open economy effect is through the parameter gamma on the IS equation (CCG1.A), where gamma enters through the inverse of \(\sigma_0\). Varying gamma between 0 and 1 causes \(\sigma_0\) to vary between \(\sigma\), the degree of consumption preference, and 1. The degree of consumption
preference enters household utility as a negative log, and hence is bounded from being too large by the assumption that utility is positively related to consumption. If \( \sigma \) happens to be very small, a large home country relative to the foreign country \( \gamma \) small) will cause inflation to dominate the definition of output. A larger value of \( \sigma \) (hardly any utility from consumption) will instead force the interaction of inflation and output to be nearly unitary.

The final international channels are via the impact of foreign output on the long run real interest rate (CCG1.B) and the natural level of output (CCG1.G). The natural level of output reacts to foreign output inversely proportional to the sign of \( \kappa_0 \). As described earlier this is due to the effects of foreign output on marginal cost, where domestic output depends inversely on marginal cost. The long run real interest rate reacts directly proportional to the sign of \( \kappa_0 \), but in relation to the expectational difference in foreign output rather than to the level.

The long run real interest rate is first defined as the natural rate of interest on capital by Wicksell (1936) as

\[
\text{"a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them ... the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods."}
\]

and deserves to be expounded on further as a slight point of contention with CCG. In the model, the long run rate is defined by equation (CCG1.B). Remembering that \( \sigma_0 = \sigma - \kappa_0 \), we can redefine (CCG1.B) as

\[
\bar{r}_t = \sigma \left( E_t[\Delta \bar{y}_{t+1}] \right) + \kappa_0 \left( E_t[\Delta \bar{y}_{t+1}^f] - E_t[\Delta \bar{y}_{t+1}] \right) \quad (3.08)
\]
where $\Delta$ stands for expected change. According to Wicksell, the natural rate should arise when money is taken out of consideration. This motivates the expected gap in flexible prices, $\bar{y}_t$, in the first part of the equation. On the other hand, foreign output does not enter under the same flexible prices assumption because CCG assumes each country takes their foreign counterpart “as given.” As will be shown in the next section, this assumption causes changes in the long run real interest rate to dominate open economy effects, completely overshadowing the “preeminent” first channel of international movements in marginal cost.

The open economy effects in T85 are more standard, but operate through largely the same channels. If foreign prices, denominated in domestic currency, are higher than domestic prices, there will be a greater demand for domestic goods, which increases output. Similarly, higher foreign output leads to increased domestic output through consumption risk sharing. The parameter $\theta$ in (T851.B) determines the openness of prices. By setting $\theta = 1$, one can shut off open economy price effects, giving a constant relation between wages and prices. On the other hand, setting $\theta = 0$ assumes purchasing power parity. In this regard, T85 differs from CCG, where purchasing power parity is assumed throughout. As is discussed subsequently, the ability to alter the openness of prices to open economy effects will make T85 an attractive model for robustness testing.

**III. 3 Policy Rules**

Both models are closed by assuming a policy rule for the central bank. Since inflation is defined solely as the change in price level and does not enter explicitly into the core equations governing output and prices under T85, any policy rule based on inflation causes the model to become indeterminate. Under a rule that depends only on inflation, the prescribed interest rate adjustment would be identical for a continuum of similarly sized jumps in prices and hence the policy does not uniquely determine a
path for the economy. In contrast, policy in CCG must be based on the inflation rate as prices are solved out of the final functional form of the model. Hence it would seem that there is no way to test the same policy rule in both T85 and CCG. I discuss in Appendix (A5) how one can modify T85 to help deal with this quandary.
IV. THE ECONOMICS BEHIND AN EXOGENOUS SHOCK

It will be illustrative to define the economic meaning of the shock term in either model before delving into the effects of exogenous shocks on the economy. It is shown in Clarida, Gali and Gertler (2002) that the exogenous term, $\mu_t$, can be thought of as an exogenous wage markup:

$$\mu_t = \frac{\delta}{(\eta_t-1)} \quad (4.01)$$

where $\eta_t$ is elasticity of labor demand and

$$\frac{W_t(h)}{P_t} = (1 + \mu_t) N_t(h)^\phi C_t^\sigma \quad (4.02)$$

gives the real wage demanded by household $h$ as a function of labor, $N_t$, and consumption, $C_t$. A higher elasticity of labor demand would lead to a lower wage markup as firms are more sensitive to wage adjustments. The authors abstract from the underlying definition of $\mu_t$ and simply define it as a proxy for workers' "power" in the labor market. More market power lets workers set a higher wage markup, forcing firms to take the new wage as given. It should be noted that Clarida Gali and Gertler introduce this exogenous wage markup term as a compromise for not including endogenous wage rigidity into their model. Unanticipated shocks to $\mu_t$ should produce shifts in real wages at least qualitatively similar to having nominal rigidities.

As shown in the nominal rigidities section, the stochastic term introduced to the contract wage equation (T851.B) in T85, can be thought of as having a similar effect on the economy. In fact, the
terms can even be considered economically similar as a positive shock in T85 corresponds to an
unexpected increase in contract wages, which increases the overall price level through an increase in
the average wage. It must be noted, though, that due to coefficients, neither shock can be considered
exactly similar in terms of magnitude. The term $\mu_t$ enters CCG as $u_t = \delta \mu_t$. And, after the
necessary manipulations are performed to arrive at a Phillips equation similar to CCG, the exogenous
shock in T85 incorporates expectational error terms that are not part of the original uncertainty.  

Tracing through a shift in $\mu_t$ for CCG gives us the following story. A completely
unanticipated, one period shift in the wage markup will raise the marginal cost of production for firms.
Since capital mobility is not modeled into CCG, the increase in marginal cost will be one for one with
the increase in wages. But, because only a fraction of firms will be able to adjust at the period of
increased wages, the subsequent increase in price level will be less than the increase in marginal cost
and, hence, inflation will increase less than one-for-one to marginal cost.  

Since shocks follow a first order, auto-regressive process,

$$\mu_t = \rho \mu_{t-1} + \epsilon \quad (4.03)$$

the unanticipated shock will also raise expectations of inflation. The central bank combats inflation by
raising nominal interest rates higher than inflation, according to:

$$i_t = \alpha E_t[\pi_{t+1}] \quad (4.04)$$

The standard trade-off ensues as this increases real interest rates, making it harder for firms to finance
production, decreasing output today.

---

22 See Appendix (A.1) for derivation.

23 Marginal cost enters the inflation equation with a coefficient of $\delta = (1-\theta)(1-\beta \theta)/\theta$ where $\theta$ is the fraction of firms adjusting prices each period, and $\beta$ is the discount factor. For the base case of $\theta = .5$, $\beta = .99$, we get $\delta = 0.51 < 1$.

24 $\alpha = 1.2$ in the initial model.
Figure 4.1. (A) Response of Country 1 to a 1% Cost Shock in Country 1.  
(B) Response of Country 2 to a 1% Cost Shock in Country 2  
Model: CCG. Both countries follow an unoptimized policy rule of the form: $i_t = 1.1 * E_t [\pi_{t+1}]$
The foreign economy (hereafter Country 2) is in turn subject to the three open economy effects discussed in (III.2). Since the two countries are symmetric, we have that $\kappa_0^f = -0.4$, the terms of trade effect of Country 1 output on Country 2 marginal cost dominates. In other words, marginal cost is expected to rise in Country 2 in reaction to the decrease of the output gap in Country 1. This is further strengthened by Figure (4.2), which shows the terms of trade drop predicted in the analysis. Yet the plots of Country 2's response to a cost shock show a contradictory fall in marginal cost (Figure 4.1A). The fall in Country 2 output is larger than predicted, thus the fall in marginal cost due to Country 2 (domestic) output falling is larger than the rise due to Country 1 (foreign) output falling. We conclude that the terms of trade effect must be dominated by other open economy disturbances.

\[ S_t = \frac{P_{f,t}}{P_{h,t}} \] in non-linearized terms. Clarida Gali and Gerler (2002) show terms of trade also satisfies the equivalent (log-linear) relation, \[ s_t = y_t - y^f_t \], which is plotted in Figure (4.2).
Figure (4.3) shows the long run natural rate of interest for Country 2 falling approximately three times as much as for Country 1. As described earlier, this is due to the definition, (CCG1.B), incorporating the actual level of Country 1 output rather than the flexible prices level, as would be expected following the strict definition by Wicksell. The larger than expected fall in output, and this the unexpected reaction of marginal cost, is thus due to the fall in the natural rate of interest. If, instead, we define the natural rate for Country 2 by,

$$\bar{r}_t^f = \sigma_0^f (E_t [\bar{y}_{t+1}^f] - \bar{y}_t^f) + \kappa_0^f (E_t [\bar{y}_{t+1}] - \bar{y}_t)$$  (4.05)

replacing Country 1 output with the flexible prices level, the change in the natural rate drop and reverses direction as shown by Figure (4.4). Under this scenario, the foreign output gap actually increases, causing marginal cost to rise as theorized. Furthermore, setting the natural rate identically to $$\bar{r}_t^f \equiv 0$$ negates any shift in foreign marginal cost. It would seem that, under the baseline coefficients assumed in Table (3.1), open economy effects are dominated by the natural rate of interest.

Figure 4.3. Natural Real Rate of Interest Change in Response to 1% Cost Shock to Country 1 in CCG
Finally, we focus on the case without the natural rate of interest. Recall that the overall magnitude of the change in Country 2 marginal cost is governed by the elasticities, $\kappa_f$ and $\kappa_0 f$, both of which depend on the open economy. Since there is no change in marginal cost, and the expected drop in terms of trade still occurs, it follows that the other open economy effect, the change in marginal rate of substitution between consumption and leisure, fully cancels out the total terms of trade effect, given by the effect of the terms of trade of both domestic and foreign output. We see that overall, cost shocks to the domestic economy do not seem to be propagated internationally, other than through the natural rate of interest. Whether this is a characteristic of CCG or an assumption embedded in the baseline coefficients will be analyzed by varying coefficients in a later section.

**Figure 4.4.** Response of Country 2 to a 1% Cost Shock in Country 1 when the real rate is of the form (4.05) under CCG
Figure 4.5. (A) Response of Country 1 to a 1% Cost Shock in Country 1. (B) Response of Country 2 to a 1% Cost Shock in Country 2.

Model: T85. Both countries follow an unoptimized policy rule of the form: \( i_t = 1.2 \times p_t \).
Contrasting the effect of a cost shock in T85 brings about a few interesting results. Figure (4.5A) shows the effect of a 1% shock on Country 1, where the exogenous term is $u_t$, a part of the contract wage equation in T85. In general a country under T85 responds similarly to CCG in the face of a cost shock. Although we cannot assume any sort of microeconomic intuition, prices increase due to the increase in wages, this causes the central bank to raise nominal rates, creating a similar fall in output. An interesting difference is that interest rates show little change in T85. While the cost shock had a level effect on prices, it did not change their growth rate.

For one possible explanation of the discrepancy, note that by Appendix (A.1), a cost shock in CCG can be defined in terms of T85 variables as $\eta_t = u_t + u_{t-1} + n_p + n_y$ where $u_t$ is the original shock to contract wages, and $n_p = (E_t - 1)[p_t - p_t]$ and $n_y = (E_t - 1)[y_t - y_t]$ can be thought of as the error agents make in forecasting prices, and output, respectively. Thus a similar shock to the stochastic terms in CCG and T85 cannot be thought of as economically equivalent. To help illustrate this point, figure (4.6) shows the response of Country 1 under a 1% to the expanded shock term, $\eta_t$, under the modified version of T85. Inflation now rises as in CCG, presumably because there is now a positive forecasting error in prices.

Open economy effects can best be explained by the change in exchange rates. Since Country 1 output falls by more than Country 2 output, global demand causes a depreciation in the exchange rate, raising Country 2 price level. As a result, the foreign central bank must also raise nominal rates. This international effect on prices does not seem to be present in CCG.

The equivalent explanation for this would be that in Appendix (A.1), it is assumed prices are a constant markup over wages, $p_t = w_t$, or, equivalently that $\theta = 1$ in (T851.B). This corresponds economically to closing prices to international interaction. As shown in figure (4.5C), setting $\theta = 1$
in T85 closes foreign variation in output, much like in CCG. Although not pursued further in this paper, this result suggests that the current definition of nominal rigidities under Calvo may not extend flawlessly to the open economy.

Figure 4.6. Response of Country 1 to a 1% Exogenous Shock of the form $\eta_t = u_t + u_{t-1} + n_t^p + n_t^r$ in T85
From an initial comparison, it seems that, while CCG compares roughly to T85 for domestic reactions to a deterministic cost shock, the open economy effects in the two models differ. If one can assume that such effects are present because T85 and CCG incorporate different aspects of interaction between Country 1 and Country 2, then testing any results from CCG under T85 will be doubly important. The gains from coordination can be assessed under two different representations of the open economy.
V. OPEN ECONOMY EFFECTS IN CCG

The following table shows a summary of how target variables fluctuate as individual model coefficients in CCG (2003) are varied, with the remaining coefficients held constant. In each case, the coefficient is tested at three separate values: a lower bound, and upper bound, and a value in between to shed some light on how the target variables transition. The target variables, inflation and output, are expressed as squared sums of deviations, normalized to reflect percentage deviations from steady state.

For instance, the target $\pi_t$ is given by, $100 \sum_{t=0}^{100} \pi_t^2$. The policy rule used is the optimal non-coordinated policy under discretion, (2.04).

For each test, we assume Country 1 experiences a 1% deterministic, and completely unexpected cost shock in the first period. For each entry, the numbers in parentheses correspond to the value of the coefficient tested (lower bound, upper bound, or middle) and the italicized value underneath is the difference between the highest and lowest results.

A few results immediately present themselves from the above analysis. Since the model is linearized around equilibrium growth rates, we can infer that, in equilibrium, the expectation of both $\pi_t$ and $y_t$ is 0. This gives the convenient result that we can treat the sum of squared deviations as a proxy for the overall, 100 period variance induced by a cost shock. Of course, this is not equal to the actual variance of the target variables since we only consider a single, 1 period deterministic shock, but it nonetheless is a useful metric to compare the effects of altering model parameters.

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26 I remind the reader that all numerical tests are performed with a set period length of 100. Examining longer and shorter period lengths produced no significant changes in results.
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<th>Squared Deviation of Targets</th>
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Table 5.1: Squared Deviation from Trend of Domestic Variables in CCG as Exogenous Coefficients are Varied.

Although, in general, a 1% cost shock does not produce more than a 1% total change in variability,\textsuperscript{27} there is a large discrepancy between the effects of different coefficients. The largest change in squared deviations of inflation comes from varying $\theta$ and $\rho$, the determinate of price stickiness and shock persistence, respectfully. Increasing the probability that firms can adjust prices (lowering $\theta$) increases the squared deviation of domestic inflation. This is interesting as one would assume that more flexible prices would actually decrease the impact of shocks on the target variables and not the other way around. If firms were able to adjust prices more frequently, the overall price level should adjust quicker, decreasing the duration and impact of shocks on the economy. Walsh hints as to

\textsuperscript{27} Remember to put in footnote on how we get more than a 1% change in variability from above data (divide by 100, sq root to get std dev, multiply by 100)
why the opposite may be true under Calvo-type price rigidity. Since we can regard each time that a firm can change prices as arrival times of a Poisson process with rate \( (1-\theta) \), the time between price optimizing opportunities increases with \( \theta \). Holding expected future inflation constant, this decreases the impact of output today on current inflation, as can be seen by the definition of \( \lambda \) and equation (CCG1.C). One can view this as a loosening of the inflation output trade-off, giving the central bank more freedom to adjust nominal rates, and decreasing the variability of both inflation and output.

On the other hand, the effect of \( \rho \) on inflation is completely expected. Increasing the persistence of the cost shock increases the total squared deviation of inflation as the aftereffects of the original 1 period shock are now present for a longer time.

The impact of varying \( \xi \), the price elasticity of intermediate good demand, is significant in changing the variability of output, but less so for altering inflation. Multiple factors seem to be at work here. CCG note that it would be plausible to assume an increase in \( \xi \) increases the costs of price dispersion and hence would cause greater inflation variability in the face of shocks. But if we also consider equation (2.04), the non-coordinated loss function faced by each country, we notice that the weight a central bank places on output variability, \( \alpha \), depends inversely on \( \xi \). Hence an increasing elasticity causes the optimizing, discretionary bank to focus less on output variability, and thereby, more on inflation variability. Combining the two effects seems to result in a central bank combating inflation slightly more than one-for-one with the impact of \( \xi \), and focusing much less on stabilizing output.

I do not present the effects of varying the above coefficients on Country 2 because all the squared variations are on the order of \( 10^{-40} \), essentially 0. Since the cost shock is assumed to originate in Country 1, the above result shows that the foreign country is almost perfectly isolated from shocks abroad. Having two identical countries immediately implies that Country 1 is also impervious
to foreign disturbances. This motivates the following proposition:

**Proposition 5.1:** In the CCG Two Country Model, there is no spillover of domestic cost shocks to the foreign economy when both central banks operate under discretionary, non-cooperative rules. Furthermore, no single variation of an exogenous parameter, within its logical boundary, can introduce spillovers.

The following table calculates the same squared deviations as above, but now we consider each country as acting under coordination. To be clear, this is achieved by substituting (2.06), the optimal discretionary policy under coordination, in place of the policy rule for both Country 1 & 2. All other parameters of the test are the same.

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28 This is also verified numerically. All changes in squared deviation are perfectly translated when the cost shock is assumed to originate in Country 2. The only difference is by the definition of \( \gamma \) as the relative size difference, varying it produces the exact opposite effects, but of the same magnitude in Country 2.
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</tr>
<tr>
<td></td>
<td>.0591</td>
</tr>
</tbody>
</table>

Table 5.2: Squared Deviation from Trend of Domestic Variables in CCG as Exogenous Coefficients are Varied when Policy is Coordinated.

The numbers in the table above are nearly identical to table (5.1). There is slightly more variability introduced when varying the parameters \( \sigma \), \( \phi \) and \( \xi \), but it is not clear whether these differences are significant, or even meaningful. As for the rest of the coefficients, the choice to coordinate has neither an effect on the direction of the change in variability, nor the magnitude. On the whole, applying a coordinated policy does not affect the squared deviation of target variables up to two significant figures.

The results suggest that target variables of the central bank do not alter in coordinating policy when the shock is assumed to originate in the domestic country. This is not completely surprising. For a domestic shock to have different effects on the home economy under coordination, the shock would not only have to alter the state of the foreign economy through international channels, but would also need
to be registered by the domestic central bank as a serious enough international disturbance to further alter domestic conditions. Such second-order disturbances would be difficult to instrument in CCG since the largest international effect, that of foreign output on domestic marginal cost, does not turn out to be very significant, as discussed in section (3.3).

The above result also has the practical intuition that shocks to the real wage do not produce significant enough effects to reverberate through the international economy. We may see some international effects of a domestic wage mark-up, but they would not be serious enough to alter policy at home any further than to combat the domestic effects, even under coordination. It is interesting to note that historically one of the only disturbances that have come close to such extreme international effects are financial sector disturbances. For instance, during the recent credit crisis, it would be plausible to consider that the Federal Reserve altered policy because of the magnitude of liquidity effects on financial institutions in Europe. In addition, Sutherland (2004) finds that in his own variant of the New Keynesian model, varying the characteristics of the international financial sector may create the sort of incentives to coordinate described above. Unfortunately, testing similar theories is out of the scope of this paper, as the CCG (2003) model assumes complete financial markets, and full risk sharing.

Although domestic disturbances do not have second-order international effects on domestic variables, it is still plausible that first-order international effects exist. I define the first-order international effects as any variability in the foreign economy introduced by a domestic shock. Since these affects are 0 for the non-coordinated case described above, any variability found under coordination can be fully attributed to the change in policy regime. The next table shows the results of varying domestic parameters and a domestic cost shock on the squared deviation of foreign variables, under coordination.
<table>
<thead>
<tr>
<th>Parameter Varied</th>
<th>Squared Deviation of Targets</th>
<th>$\pi_i^f$</th>
<th>$y_i^f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma \in [0,5,1]$</td>
<td>{1.56E-005 , 3.40E-006 , 0.00E+000}</td>
<td>1.56E-005</td>
<td>1.73E-004</td>
</tr>
<tr>
<td>$\phi \in [-.9,1,100]$</td>
<td>{9.40E-006 , 3.40E-006 , 5.36E-011}</td>
<td>9.40E-006</td>
<td>1.20E-003</td>
</tr>
<tr>
<td>$\gamma \in [.01,.5,99]$</td>
<td>{1.33E-005 , 3.40E-006 , 1.36E-009}</td>
<td>1.33E-005</td>
<td>8.21E-005</td>
</tr>
<tr>
<td>$\beta \in [.01,.5,99]$</td>
<td>{4.81E-007 , 5.61E-007 , 3.40E-006}</td>
<td>2.92E-006</td>
<td>1.45E-005</td>
</tr>
<tr>
<td>$\theta \in [.1,.5,99]$</td>
<td>{.0013 , 5.54E-004 , 6.46E-013}</td>
<td>1.30E-003</td>
<td>2.22E-006</td>
</tr>
<tr>
<td>$\xi \in [.01,1,100]$</td>
<td>{8.57E-010 , 3.40E-006 , 6.83E-008}</td>
<td>6.74E-008</td>
<td>2.60E-006</td>
</tr>
<tr>
<td>$\rho \in [.01,.5,99]$</td>
<td>{1.29E-011 , 2.34E-010 , 4.12E-005}</td>
<td>4.12E-005</td>
<td>1.53E-004</td>
</tr>
</tbody>
</table>

Table 5.3 Squared Deviation from Trend of Foreign Variables in Response to a Domestic Shock in CCG as Exogenous Coefficients are Varied when Policy is Coordinated.

Under cooperation, international effects are larger compared to the non-cooperative case, where all effects were practically 0. Unfortunately, the impact on foreign squared deviation is still not very significant. The single largest foreign impact stems from setting $\phi = -.9$, which corresponds to the highly unrealistic case of almost 0 dis-utility of labor for the representative household. Intuitively, output variability increases more abroad because the foreign central bank needs to change interest rates more in order to compensate for workers having less dis-utility of labor. The effect on foreign output is further exacerbated because workers abroad equivalently have almost no dis-utility of labor. The change in foreign output variability ends up being about one fourth what the variability would be if the shock originated in country 2. The impact completely disappears when $\phi$ is set at more reasonable
levels.

All other international effects under coordination are at least one order of magnitude less than the corresponding domestic change. In fact, as Table (5.4) shows, the majority of international effects on squared deviation are smaller on an order of magnitude 4 or more than the corresponding domestic case. In the face of domestic disturbances, any one period international shock would hardly be noticeable at home.

<table>
<thead>
<tr>
<th>Order of Magnitude Difference</th>
<th>Number of Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4+</td>
<td>24</td>
</tr>
</tbody>
</table>

*Table 5.4 Order of Magnitude Difference in Domestic and Foreign Variation*

Again, we present the theoretical argument that, since Country 1 and Country 2 are identical, shocks originating abroad would have the same, minimal, first and second order effects on the global economy. Combining the above results, we have an analogue to Proposition x1 under coordinated policy.

**Proposition 5.2:** Even under coordinated policy, the impact of domestic disturbances abroad is minimal under a discretionary regime. Both the first-order international spillover, and the second-order response of foreign agencies to help mitigate global welfare losses are small

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29 Difference between Domestic and International effects under same coefficients. Orders of Magnitude are rounded.
relative to domestic effects. These results are found to be robust under all allowable parameter variations, with the exception of one extreme case described above.

I emphasize that all the above results were obtained under the assumption of discretionary policy, the mathematical implications of which are described in an earlier section. CCG themselves note that both qualitative and quantitative results may differ under different policy regimes. Hence, in order to differentiate the method of finding an optimal policy from the act of coordination, and to be able to compare loss results on a relative scale, I introduce an alternative approach to optimal policy – the simple rule.
VI. THE SIMPLE RULE APPROACH TO OPTIMAL POLICY

The general approach to finding a “simple” policy rule is described in Walsh (2003, pp 529-31). Before optimizing a predetermined objective function, a central bank makes the assumption that the best it can hope to achieve is a policy rule that minimizes the effect economic disturbances can have on target variable deviations from trend. In the CCG model, this is mathematically equivalent to minimizing the coefficients $b_y$ and $b_n$ in the following relations:

$$y_t = b_y u_t \quad (6.01)$$
$$\pi_t = b_n u_t \quad (6.02)$$

for all time, to have the lowest loss possible subject to the constraints in the economy. Committing to a rule can be thought of as a reasonable middle ground between optimal commitment and discretion policies. It carries some of the advantages of both commitment and discretion while also alleviating potential problems encountered with either of the extreme approaches.

Optimal commitment, which stipulates how the central bank will react in every period in the future, is ideal in the sense it gives perfect information about the future actions of a central bank, eliminating expectational uncertainty. On the other hand, it is inconceivable that any central bank would have the technology or knowledge necessary to be able to be able to compensate for all possible events in the future. Also, the optimal mathematical solution under commitment, derived in Walsh (2003 pp 524) and Woodford (1999a), shows temporal inconsistency. Broadly speaking, the solution implies different relations between policy variables at the current time $t$, and for future periods. The problem is that under this solution, a central bank will find it impossible to commit. At period $t$, the bank will perform one action and then propose its actions for all future periods, but at period $t+1$ it will renege on his earlier promise in favor of the current period prescription. Hence optimal commitment
can be likened to a Markov Chain, where at each period, the past history of policy action becomes irrelevant.\textsuperscript{30}

In contrast, optimal discretionary policy does not require the central bank to have any credibility or commitment technology and there are never inconsistency issues because consistency is never expected in the first place. A problem is that agents never know how a central bank will react. Especially in the event of an unanticipated, exogenous shock, this creates expectational uncertainty. Such a policy is likely to be sub-optimal in the set of all policy types as central banks have no control over expectations of future inflation or output, and hence are limited in the tools they can employ.

Commitment to a rule should eliminate some of the issues described above. All that is needed to fully describe a rule are the values of the coefficients $b_y$ and $b_n$ and the stochastic characterization of the shocks affecting the system. Although the latter requirement may be difficult to satisfy in practice, a model such as CCG has this information readily available. The rule eliminates the inconsistency problem of commitment as the prescription is the same for all periods (the coefficients are assumed to not depend on time) and the expectational uncertainty problem of discretion.

Commitment to a rule should be at least as optimal as discretion almost by definition. The equations bearing the optimal levels of inflation and output under discretion,

\begin{align}
y_t &= -\xi\psi u_t \quad (6.03) \\
\pi_t &= \psi u_t \quad (6.04)\textsuperscript{31}
\end{align}

are functionally identical to the assumed solutions under a simple rule, (6.01) and (6.02). By definition,

\textsuperscript{30} Both Woodford (1999a) and Nelson (2000a) discuss the “timeless perspective” approach to commitment policy where a central bank assumes they are in an arbitrary time period far from the initial period optimization. This eliminates the temporal inconsistency, but also derives a result that is not necessarily the same as the strict optimization under commitment. It is not clear whether or not timeless perspective policies are dominated strategies.

\textsuperscript{31} From Clarida, Gali and Gertler (2002). Definitions of the coefficients $\%xi$ and $\%phi$ can be found in table (3.2).
$b_y$ and $b_\pi$ are the coefficients that minimize this functional relation. Hence they must be at least as effective in reducing variation as the coefficients under optimal discretion.

As shown in Appendix (A4), the minimum values of the coefficients for CCG are given by:

$$b_y = \frac{-\xi}{\lambda \xi + (1 - \beta \rho)^2} \quad (6.05)$$

$$b_\pi = \frac{(1 - \beta \rho)}{\lambda \xi + (1 - \beta \rho)^2} \quad (6.06)$$

Substituting these values into the relations (6.01) and (6.01) and then solving the IS curve (CCG1.A) gives the following policy that satisfies optimal commitment to a simple rule,

$$i_t = \left[ 1 + \frac{\sigma_0 \xi (1 - \rho)}{(1 - \beta \rho) \rho} \right] E_t \left[ \pi_{t+1} \right] + \bar{r}_t \quad (6.07).$$

We have assumed above that the central bank solving for the optimal simple rule does not pursue international coordination. Although combining derivations in the appendix with the coordination calculations in CCG (2003) provides a straightforward road map for calculating the optimal simple rule under coordination, this paper does not pursue this route.
VII. NUMERICAL ANALYSIS OF CCG

With the three policies defined, it is time to compare their implications in CCG. The following table shows the results of the loss functions (2.04) and (2.06) under the three different optimal policy rules and two separate shocks. The shocks are differentiated by their origin. A shock origin of $C1$ implies shock to Country 1 only. Subsequently, an origin of $C1&2$ implies a shock to both countries simultaneously. The third case, of a shock originating in Country 2 is not considered, again, due to our symmetry assumption. In an attempt at statistical rigor, I model stochastic simulations of CCG as described in the literature review. This corresponds to drawing a random representation of the cost shock given a specified mean and variance. The results of 100 stochastic simulations under each optimal policy rule are summarized in Table (7.1). In each cell, the mean and standard deviation (in parentheses) is represented for the corresponding loss function.

<table>
<thead>
<tr>
<th>Policy Rule</th>
<th>Shock Origin</th>
<th>Non-Coordinated Loss</th>
<th>Coordinated Loss Result</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Country 1</td>
<td>Country 2</td>
<td></td>
</tr>
<tr>
<td>Discretion, No Coordination</td>
<td>$C1$</td>
<td>1.97E-002 (1.28E-002)</td>
<td>1.22E-030 (8.04E-031)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.97E-003 (6.51E-003)</td>
<td></td>
</tr>
<tr>
<td>Discretion, Coordination</td>
<td>$C1$</td>
<td>1.98E-002 (1.24E-002)</td>
<td>6.74E-004 (4.22E-004)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.03E-002 (6.44E-003)</td>
<td></td>
</tr>
<tr>
<td>Simple Rule, No Coordination</td>
<td>$C1$</td>
<td>5.77E-003 (3.36E-003)</td>
<td>1.58E-034 (9.30E-035)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.07E-003 (1.79E-003)</td>
<td></td>
</tr>
<tr>
<td>Discretion, No Coordination</td>
<td>$C1&amp;2$</td>
<td>1.74E-002 (1.02E-002)</td>
<td>1.96E-002 (1.01E-002)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.87E-002 (7.09E-003)</td>
<td></td>
</tr>
<tr>
<td>Discretion, Coordination</td>
<td>$C1&amp;2$</td>
<td>1.97E-002 (1.10E-002)</td>
<td>2.19E-002 (1.41E-002)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.09E-002 (9.41E-003)</td>
<td></td>
</tr>
<tr>
<td>Simple Rule, No Coordination</td>
<td>$C1&amp;2$</td>
<td>5.35E-003 (3.13E-003)</td>
<td>5.90E-003 (3.77E-003)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.92E-003 (2.51E-003)</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.1 Results of Stochastic Simulations on CCG under Different Policy Rules
Aligning with previous analysis, the lack of coordination virtually shuts down all international effects. When the cost shock originates in Country 1, Country 2 is able to keep its loss optimally minimized, at effectively 0. This result is confirmed both when central banks operate under discretion and under a simple rule. The intuition is relatively clear. We know that other than for potential changes in the natural real interest rate, open economy effects under no-coordination interact to cancel each other out, leaving foreign marginal cost unchanged. The natural real interest rate effect, on the other hand, is completely countered by including it in the optimal policy rule. Hence foreign inflation variation is negated by the structure of open economy effects and any output variation that would have resulted from a fluctuating natural real interest rate is offset 1-for-1 by optimally setting nominal rates.

Second, when countries do not coordinate, a simple rule can produce an order of magnitude less loss as compared to an optimal policy under discretion. This shows that the optimal rule found under discretion does not fall into the set of policy rules that optimize the non-coordinated loss function, (2.04). Considering the method of derivation for each rule, the simple rule only makes assumptions on the form of the solution. Given the assumption, it can be mathematically proven that the coefficients found are optimal for all rules of this form. The optimal levels for output and inflation under discretion are functionally identical to the simple rule assumption, and the coefficients under discretion do not match those under the simple rule. Hence we can conclude that the optimal rule found under discretion does not minimize the loss function given by (2.04).  

Third, coordinating monetary policy does not lower the coordinated loss by any significant margin over the non-coordinated policy rules. This result holds both under an isolated shock originating in Country 1, and a simultaneous, world-wide shock. Since the losses under coordinated and non-coordinated discretionary policy are similar it is difficult to say whether the coordinated policy actually

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32 See section (x.x)
33 Of course, it is possible that there exist some other functional form that minimizes loss even further than the simple rule.
does worse than the non-coordinated policy. The large discrepancy between coordination and the simple rule, though, signifies that it is possible to find non-coordinated policies that give greater welfare gains (smaller losses) than the coordination policy above.

The theories above are also tested for statistical significance by conducting standard 2 sample t-tests to compare the mean loss under different policies. Table 7.2 contains the results. Each test compared the loss results under two policies for the null hypothesis that the means of each were equal. The summarized results show the alternative hypothesis for each test. In each cell, the first number gives the probability that we reject the null in favor of the alternative hypothesis and the second number is the t statistic. All tests had 198 degrees of freedom given that each sample had 100 entries. A single star, * implies significance at the 90% confidence level, while two stars ** imply significance at 95% confidence.

<table>
<thead>
<tr>
<th>Shock</th>
<th>Alternative Hypothesis</th>
<th>Loss Results Country 1</th>
<th>Loss Results Country 2</th>
<th>Loss Results Coordination</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>SR &lt; DNC</td>
<td>1**</td>
<td>1**</td>
<td>1**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t = 10.4691</td>
<td>t = 15.1904</td>
<td>t = 10.2116</td>
</tr>
<tr>
<td></td>
<td>SR &lt; DC</td>
<td>n/a</td>
<td></td>
<td>1**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>t = 10.7959</td>
</tr>
<tr>
<td></td>
<td>DNC &lt;&gt; DC</td>
<td></td>
<td></td>
<td>0.7236</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>t = -0.3542</td>
</tr>
<tr>
<td>C1&amp;C2</td>
<td>SR &lt; DNC</td>
<td>1**</td>
<td>1**</td>
<td>1**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t = 11.2813</td>
<td>t = 12.6629</td>
<td>t = 16.9822</td>
</tr>
<tr>
<td></td>
<td>SR &lt; DC</td>
<td>n/a</td>
<td></td>
<td>1**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>t = 15.4090</td>
</tr>
<tr>
<td></td>
<td>DNC &lt;&gt; DC</td>
<td></td>
<td></td>
<td>0.0597</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>t = -1.8939</td>
</tr>
</tbody>
</table>

Table 7.2 Results of Statistical Analysis on Stochastic Simulations in CCG

Both our theorized results above are supported in Table 7.2. The simple rule has a statistically
lower average loss as compared to both discretionary rules – coordination and non-coordination. Under both types of shock origin considered, I reject the null hypothesis that the loss results under a simple rule and discretionary policy are equal. At the same time, a similar null hypothesis for discretionary policy under coordination and non-coordination cannot be rejected. Thus we come to the following conclusion:

**Proposition 7.1:** In the baseline version of CCG, operating under coordinated discretionary policy is a strictly dominated strategy.

It is not altogether apparent why a policy optimized under a certain objective function does not, in fact, optimize that objective. Some of the potential causes were identified in the discussions above. It may be that taking all expected variables as given does not provide a good approximation of an optimal policy. It may also be that the lack of open economy interactions does not offer any gain from optimizing global welfare. The optimal coefficient on the foreign inflation term under coordination, $\theta_2 = -0.008$, gives evidence towards the second theory.

Finally, the policies are analyzed under an independent metric – the second moments of output and inflation. Using the equations for optimal output and inflation, (6.01) through (6.04) we can calculate what analytical variances should be under non-coordinated discretion and simple rule policies as functions of the cost shock variance. The results are in Table (7.3). Note that we cannot calculate similar functions for the coordinated case as there are too many unknown variables.\(^{34}\)

\(^{34}\) Clarida, Gali and Gertler (2002) manage to calculate optimal levels of $\pi_t$ and $\pi^f_t$ as functions of the domestic and foreign cost shocks only, but the simplifying assumptions that they have to make in order to do so render them not very useful for numerical analysis. They also do not match with the approximated moments calculated later on. The relations (and subsequent output calculations) are included in Appendix (x.x) for completeness.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Equation</th>
<th>Baseline Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Discretion, No Coordination</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Country 1</strong></td>
<td>$\sigma^2_y$</td>
<td>$(\xi \psi)^2 \sigma^2_{u(t,C1)}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma^2_\pi$</td>
<td>$\psi^2 \sigma^2_{u(t,C1)}$</td>
</tr>
<tr>
<td><strong>Country 2</strong></td>
<td>$\sigma^2_{y(F)}$</td>
<td>$(\xi \psi)^2 \sigma^2_{u(t,C2)}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma^2_{\pi(F)}$</td>
<td>$\psi^2 \sigma^2_{u(t,C2)}$</td>
</tr>
<tr>
<td><strong>Simple Rule, No Coordination</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Country 1</strong></td>
<td>$\sigma^2_y$</td>
<td>$\left(\frac{\xi}{(\lambda \xi + (1-\beta \rho)^2)}\right)^2 \sigma^2_{u(t,C1)}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma^2_\pi$</td>
<td>$\left(\frac{1-\beta \rho}{(\lambda \xi + (1-\beta \rho)^2)}\right)^2 \sigma^2_{u(t,C1)}$</td>
</tr>
<tr>
<td><strong>Country 2</strong></td>
<td>$\sigma^2_{y(F)}$</td>
<td>$\left(\frac{\xi}{(\lambda f \xi + (1-\beta \rho)^2)}\right)^2 \sigma^2_{u(t,C2)}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma^2_{\pi(F)}$</td>
<td>$\left(\frac{1-\beta \rho}{(\lambda f \xi + (1-\beta \rho)^2)}\right)^2 \sigma^2_{u(t,C2)}$</td>
</tr>
</tbody>
</table>

Table 7.3 Variances Implied by Optimal Rules

It seems as though the simple rule works by lowering the variance of inflation in spite of output variance. We can attribute this action to two possible factors: (1) that the weight on inflation is higher in the loss function (2.04) and (2) that under CCG, it costs less in terms of output variance to lower inflation variance than visa versa. The first factor holds as $\alpha=0.27$ in the baseline model, corresponding to approximately an 80/20 weight on minimizing inflation to output. For the second, we calculate the theoretical variance. The numerical method used in these calculations is explained in section (3.3).

From Tables (7.4) and (7.5) we see that the simple rule does optimize over the discretion rule by
lowering the variance of inflation, giving up output stability in turn. This result holds whether considering the output gap under domestically or globally flexible prices (y_t versus ŷ_t ) and under both a one sided shock and a simultaneous, symmetric shock. Since the difference in loss from lowering inflation variance is greater than the difference in raising output variance by over two orders of magnitude, I conclude that,

**Proposition 7.2:** Under the baseline CCG, factor (2) holds, e.g. the a rate of substitution between inflation variance and output variance is \( < -1 \).

<table>
<thead>
<tr>
<th>Variance</th>
<th>( \sigma^2(u_i) = 0.01 )</th>
<th>( \sigma^2(u_i') = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule</td>
<td>( y_t )</td>
<td>( \pi_t )</td>
</tr>
<tr>
<td>Discretion, No Coordination</td>
<td>2.67E-004</td>
<td>2.67E-004</td>
</tr>
<tr>
<td>Discretion, Coordination</td>
<td>2.62E-004</td>
<td>2.91E-004</td>
</tr>
<tr>
<td>Simple Rule, No Coordination</td>
<td>3.90E-004</td>
<td>1.38E-006</td>
</tr>
</tbody>
</table>

*Table 7.4 Theoretical Variances for CCG Under a Cost Shock Originating in Country 1*

<table>
<thead>
<tr>
<th>Variance</th>
<th>( \sigma^2(u_i) = 0.01 )</th>
<th>( \sigma^2(u_i') = 0.01 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule</td>
<td>( y_t )</td>
<td>( \pi_t )</td>
</tr>
<tr>
<td>Discretion, No Coordination</td>
<td>2.67E-004</td>
<td>2.67E-004</td>
</tr>
<tr>
<td>Discretion, Coordination</td>
<td>2.63E-004</td>
<td>3.03E-004</td>
</tr>
<tr>
<td>Simple Rule, No Coordination</td>
<td>3.90E-004</td>
<td>1.38E-006</td>
</tr>
</tbody>
</table>

*Table 7.5 Theoretical Variances for CCG Under a Cost Shock Originating in Countries 1 and 2*
The variance tables also give us another look at the effects of coordination on CCG. By tables (7.4) and (7.5) a coordinated policy does lower the variance of the output gap under globally flexible prices, as the policy is designed to do. On the other hand, coordination raises inflation variance globally. Since the loss function is designed to weigh inflation variance more heavily than output variance, this result further suggests a lack of gains from coordination.

Following McCallum (1999), I now test the robustness of my results by repeating the above analysis in T85.

**VIII. ROBUSTNESS TESTING IN T85**

The tables below represent identical calculations for the results of the loss functions (2.04) and (2.06) under the three different optimal policy rules and two separate shocks. For a description of the shocks see section 7. To further test robustness I examine T85 under both assumptions of 2 period and 3 period contract lengths, as discussed in Appendix A1 and A2. The three policies are not examined under the relaxed PPP version of T85 due to structural barriers discussed in appendix A3. Again, the tables show a result of 100 stochastic simulations under each of the three policies considered: discretionary without coordination, discretionary with coordination, and a simple rule without coordination. In each cell, the mean and standard deviation (in parentheses) is represented for the corresponding loss function.
Table 8.1 Results of Stochastic Simulations on T85 with Symmetric Two Period Contracts under Different Policy Rules

<table>
<thead>
<tr>
<th>Policy Rule</th>
<th>Shock Origin</th>
<th>Non-Coordinated Loss</th>
<th>Coordinated Loss Result</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Country 1</td>
<td>Country 2</td>
</tr>
<tr>
<td>Discretion, No Coordination</td>
<td>C1</td>
<td>6.01E-001</td>
<td>1.43E-001 (8.68E-002)</td>
</tr>
<tr>
<td>Discretion, Coordination</td>
<td>C1</td>
<td>7.83E-001</td>
<td>3.09E-001 (2.10E-001)</td>
</tr>
<tr>
<td>Simple Rule, No Coordination</td>
<td>C1</td>
<td>5.60E-003</td>
<td>1.04E-003 (7.25E-004)</td>
</tr>
<tr>
<td>Discretion, No Coordination</td>
<td>C1&amp;2</td>
<td>8.14E-001</td>
<td>7.62E-001 (3.77E-001)</td>
</tr>
<tr>
<td>Discretion, Coordination</td>
<td>C1&amp;2</td>
<td>1.12E+000</td>
<td>1.10E+000 (5.98E-001)</td>
</tr>
<tr>
<td>Simple Rule, No Coordination</td>
<td>C1&amp;2</td>
<td>5.97E-003</td>
<td>6.39E-003 (3.53E-003)</td>
</tr>
</tbody>
</table>

Table 8.1 Results of Stochastic Simulations on T85
The main results from examining loss functions under CCG can be generalized to T85 under both 2 and 3 period contracts. It is interesting to note that coordinated policy seems to perform worse in the T85 model than in CCG. A probable explanation of this is that the coefficients of policy optimized under CCG are far from optimal under T85. Adding in terms depending on foreign variables, no matter what they were, would increase variation under T85 in this case. I do not pursue the topic further, but it would be insightful to perform similar optimizing calculations on T85 in order to compare what each model prescribes as optimal policy.

Another observation is that increasing the contract length seems to decrease variability of output and inflation in T85. As discussed in section 6, this result seems at odds with initial intuition that greater nominal rigidity would cause similar shocks to have larger impacts on the economy, but can possibly be explained by a lower emphasis on the present with greater rigidity. As shown in equation (A.13), and discussed in section 3.1, increasing contract lengths to three periods imposes both inflation persistence and more emphasis on expectations in the Phillips-type equation. It also diminishes the elasticity of inflation with respect to output by a factor of \( \frac{1}{3} \).

Still, the main result is still maintained that in the face of both symmetric and asymmetric cost shocks, coordinated monetary policy does not provide relief in terms of lower losses over the other two policies. Again this result is verified statistically with a 2-sample t-test comparing the means of stochastic simulations under different loss function. As before, In each cell, the first number gives the probability that we reject the null in favor of the alternative hypothesis and the second number is the t statistic. All tests had 198 degrees of freedom given that each sample had 100 entries. A single star, * implies significance at the 90% confidence level, while two stars ** imply significance at 95%
The analysis above is largely confirmed by the statistical tests. For both types of shocks the null hypothesis that there is no difference between average loss under discretionary policy and the simple rule is rejected in favor of the alternative. These results hold for both coordinated and non-coordinated discretionary policy and are significant at a 95% confidence level. Furthermore, the results are robust under both assumptions of 2 and 3 periods symmetric contract lengths. The difference compared to CCG highlighted above is also statistically significant. In all cases, null hypothesis that coordination and non-coordination have the same average loss is rejected at the 95% confidence level. Coordination monetary policy has a statistically significant detrimental effect on welfare loss in T85. Though not able to be proven statistically, this negative effect of coordination seems to weaker with longer contract lengths.

<table>
<thead>
<tr>
<th>Shock</th>
<th>Alternative Hypothesis</th>
<th>Loss Results</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Country 1</td>
<td>Country 2</td>
<td>Coord</td>
</tr>
<tr>
<td>C1</td>
<td>SR &lt; DNC</td>
<td>**</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t = 16.4255</td>
<td>t = 16.3317</td>
<td>t = 16.4006</td>
</tr>
<tr>
<td></td>
<td>SR &lt; DC</td>
<td></td>
<td></td>
<td>**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>t = 14.7056</td>
</tr>
<tr>
<td></td>
<td>DNC &lt;&gt; DC</td>
<td></td>
<td></td>
<td>.9999**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>t = -4.0364</td>
</tr>
<tr>
<td>C1&amp;C2</td>
<td>SR &lt; DNC</td>
<td>**</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t = 12.9094</td>
<td>t = 20.0368</td>
<td>t = 17.4355</td>
</tr>
<tr>
<td></td>
<td>SR &lt; DC</td>
<td></td>
<td></td>
<td>**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>t = 17.6909</td>
</tr>
<tr>
<td></td>
<td>DNC &lt;&gt; DC</td>
<td></td>
<td></td>
<td>**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>t = -4.1797</td>
</tr>
</tbody>
</table>

Table 8.2 Results of Statistical Analysis on Stochastic Simulations in T85 with Symmetric Two Period Contract Lengths
Leonid Boris Pekelis
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May 14th, 2008

Shock | Alternative Hypothesis | Loss Results
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Country 1</td>
<td>Country 2</td>
<td>Coordination</td>
<td></td>
</tr>
<tr>
<td>---------------------------------</td>
<td>-------------------</td>
<td>-------------------</td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>SR &lt; DNC</td>
<td>**</td>
<td>t = 15.5476</td>
</tr>
<tr>
<td></td>
<td>SR &lt; DC</td>
<td>n/a</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>DNC &lt;&gt; DC</td>
<td>n/a</td>
<td>**</td>
</tr>
<tr>
<td>C1&amp;C2</td>
<td>SR &lt; DNC</td>
<td>**</td>
<td>t = 21.2518</td>
</tr>
<tr>
<td></td>
<td>SR &lt; DC</td>
<td>n/a</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>DNC &lt;&gt; DC</td>
<td>n/a</td>
<td>**</td>
</tr>
</tbody>
</table>

Table 8.3 Results of Statistical Analysis on Stochastic Simulations in T85 with Symmetric Three Period Contract Lengths

Finally, I calculate the theoretical variance of output and inflation under all the policies for T85 under symmetric contracts of 2 periods in length. The results are in table 8.4. A similar story is presented to CCG. The simple rule works by putting weight on containing inflation, while allowing for higher output variance. One interesting discontinuity is that a simple rule also seems to lower foreign output variation in the case of asymmetric shocks. Since foreign output variance is already several orders of magnitude smaller than domestic variance, and the effect is drowned out under symmetric cost shocks, the author leaves the point as merely an observation. We also see the same effect of the coordinated policy on domestic variables. Contrary to the goal of the loss function, discretionary policy under coordination lowers output variance and allows inflation variance to climb. The effect is mirrored in both countries in the face of a symmetric shock.  

Variance results for the 3 contract version of T85 are not available due to problems with calculations. The author
Table 8.4 Theoretical Variances for CCG Under a Cost Shock Originating in Country 1 and a Simultaneous, Global Cost Shock

Although the three policies in question were not optimized for T85, and no distinction between the output gap under globally or domestically flexible prices was made, the main results obtained for CCG are robust to T85.

suspects it may be a cause of lagged inflation appearing with a negative sign in the inflation equation, which invalidates the optimization technique used to minimize the variance condition in (x.x). Further analysis on how to reformulate the inflation equation in the presence of 3 contracts is left for future research.
IX. CONCLUSION

This paper rigorously analyzes the potential for welfare gains from coordinating monetary policy in the New Keynesian model of Clarida Gali and Gertler (2002). By focusing on a numerical analysis of the model, it has the advantage of being able to put concrete numbers into the objective functions of central bank. Hence it is possible not only to test whether gains from coordination exist, but also to comment on the size of the gains, and, specifically, whether potential gains can be considered significant. I show that, although the theoretical arguments in Clarida Gali and Gerler (2002) imply that gains from coordination should exist, these gains are not statistically different from 0 under actual numerical simulations.

I also show that it is simple and straightforward to find other monetary policies that are preferred to an optimized discretionary policy, regardless of the decision to coordinate. In particular, the simple rule approach to optimizing monetary policy gives lower loss results than either of the policies derived from the assumption of a discretionary central bank. These results are also statistically significant among a number of stochastic simulations. It should be noted that this result is not in contradiction to Clarida, Gali and Gertler as they themselves discuss the possibility of other optimized rules outperforming those derived to test coordination, although they never prove it outright.

One interesting fact that comes out of finding a more optimal method of optimizing policy is that it may help to explain why numerical gains were not found from coordination. The fact that the policies found under discretion are shown not to be in the set of optimal policies under a lack of coordination, uncovers the possibility that they are even farther from the set of optimal policies under coordination. In other words, it is plausible that coordination under a different regime, such as the simple rule, may be better at optimizing the coordinated objective function to the point that it could find gains from coordination where a discretionary policy could not. Although such a result would not
be completely intuitive, it is nonetheless an intriguing avenue for future research.

These results are found to be generally robust under a modification of the model used in Taylor (1985), and under the cases of both symmetric two period and three period contract lengths. One surprising result is that under T85, the coordinating policy is statistically worse at optimizing the coordinated objective over the uncoordinated policy. A likely theory is that the policy coefficients are sufficiently closer to an optimal level under CCG over T85 and therefore the extra variability inherent in allowing domestic rates to respond directly to foreign variables overshadows any possible gains to global stabilization. It would be pertinent to study how fully optimized policies under either model relate to each other.

A number of secondary results also presented themselves in the course of this paper. For instance, both the T85 and CCG models exhibit the characteristic that lowering inflation variation imposes less of a cost on output variation than visa versa. This is a characteristic that the simple rule approach incorporates, but the discretionary approach misses. The result is that, under an optimized simple rule policy, inflation is reigned in very quickly in response to an exogenous cost push shock. This has the usual trade off of increasing the resulting fall in output, but the extra loss in output is outweighed by the gain of lower inflation for the duration of the shock. Even when a central bank puts equal weight on inflation and output variation in its objective, reducing inflation variation is preferred to reducing output variation.

A final result is that the magnitude of open economy effects in CCG are shown to be very small. The result continues to hold even when all exogenously defined parameters are varied over a logical range. This stands in contract to T85, where significant open economy effects have been documented over a variety of different parameters (Taylor 1985, 1993). One potential theory to explain this result may be that firm pricing behavior in CCG is not fully optimized for open economy effects. In section (3.1), I demonstrate that in order to derive a similar Phillips-style curve to the one obtained in CCG,
prices must be assumed to be closed to open economy effects in T85. Relaxing this assumption arrives at a Phillips-style curve that incorporates expectations of foreign inflation not found under CCG. Although not tested outright, it is likely that these extra terms are the reason behind greater open economy effects under T85.

The overarching outcome of the results presented above is that there are still many questions to be answered regarding the inner workings of the open economy. A promising direction seems to be continued relaxation of assumptions and combination of relationships that seem to have significant effects in the open economy. One avenue would be to incorporate relaxed purchasing power parity, or, equivalently, the incomplete pass through of exchange rates, into a model with persistent, micro-founded nominal rigidities, as in Calvo (1983), or Taylor (1985). Both assumptions have important implications for international economics. There is no reason not to assume interactions between them as well.

Another road is continued study of international financial markets. As covered in the literature review, Sutherland (2004) shows that the structure of financial markets can have significant effects on coordination. The financial markets approach has the additional benefit of being particularly relevant in light of recent economic events. Even though the liquidity crisis of the past year originated in the US, it has had and will continue to have far reaching effects the world over precisely because financial markets are so interconnected internationally. (BBC, IMG slashes world growth forecast)

In a recent article by Engel and West, the authors argue that, while there is evidence that exchange rates have predictive power over fundamental variables such as output, prices and the terms of trade the reverse relation, of fundamental variables having predictive power over exchange rates, is non-existent. When trying to predict exchange rates given contemporary theories, models and data, it is

\[ \theta = 1 \text{ in } p_t = \theta w_t + (1-\theta)(e_t + p_t') \text{ where } p_t \text{ is prices, } w_t \text{ is the average wage and } e_t \text{ is the exchange rate} \]
found that exchange rates follow a pattern that is indistinguishable from a random walk (Engel and West, 2005). In the end, it seems as though Branson's argument still rings true today just as much as it did over two and a half decades in the past. The upside is that there is still plenty of work left to be done, both for economists and the NSF.
X. REFERENCE LIST


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XI. APPENDIX

Appendix (A1) – Phillips Curve in Taylor Model, 2 period contract lengths, closed prices

Assume that wages, once set, are unable to be changed for two periods, and an equal proportion of workers – one half – renegotiate wages each period. Hence the average wage a firms faces will be

$$w_t = \frac{1}{2} \sum_{i=0}^{1} x_{t-i} \quad (A.01)$$

since wages set one period ago will still be in effect. We now assume two further conditions. One, that prices in each country do not depend on foreign prices, and are simply set as a markup over wages today:

$$p_t = w_t \quad (A.02)$$

And two, that contract wages today (denoted $x_t$) are set based on a weighted average of wages, prices, output, and a stochastic error term that can be interpreted as any exogenous factor arising in an unexpected increase in wages,

$$x_t = \frac{\delta}{2} \sum_{i=0}^{1} w_{t+i} + \frac{(1-\delta)}{2} \sum_{i=0}^{1} p_{t+i} + \frac{\gamma}{2} \sum_{i=0}^{1} y_{t+i} + u_t \quad (A.03)$$

Above, $\delta$ represents weight given to wage persistence (a delta of 0 means workers do not take into account current or past wages when determining new contracts), $\gamma$ is the weight on output, and any

---

37 A couple of things to note. Since we assume a constant markup, we can normalize prices to drop the markup term. Also, the price equation corresponds to setting $\theta = 1$ in T85.
variables with time subscripts in the future can be regarded as expectations of those variables. The stochastic error term evolves according to

\[ u_t = \rho u_{t-1} + \epsilon_t \]  

(A.04)

where \( \epsilon_t \) is iid over time with mean 0 and variance \( \sigma^2 \). Combining the above three equations gives an expression for prices today:

\[ p_t = \frac{1}{2}(x_t + x_{t-1}) = \frac{1}{4}(p_t + E_t p_{t+1} + p_{t-1} + E_{t-1} p_t + y_t + E_t y_{t+1} + y_{t-1} + E_{t-1} y_t) + u_t + u_{t-1} \]

\[ = \frac{1}{4}(2p_t + E_t p_{t+1} + p_{t-1} + y_t + E_t y_{t+1} + y_{t-1}) + \eta_t \]  

(A.05)

since by (A.02) we have \( \delta w_t + (1-\delta) p_t = p_t \). The term \( \eta_t \) combines the stochastic shock with an expectational error term, \( \eta_t = u_t + u_{t-1} + (E_{t-1} p_t - p_t) + (E_{t-1} y_t - y_t) \). Rearranging and letting

\[ \zeta^1(y_t) = y_{t-1} + 2y_t + E_t y_{t+1} \]  

gives us

\[ p_t = \frac{1}{2}(p_{t-1} + E_t p_{t+1} + y_t \zeta^1(y_t) + \eta_t) \]  

(A.06)

Finally, we can express (A.06) in terms of inflation\(^{38}\) by

\[ \pi_t = E_t \pi_{t+1} + y \zeta^1(y_t) + \eta_t \]  

(A.07)

---

\(^{38}\) where inflation has the standard definition, \( \pi_t = p_t - p_{t-1} \)
Appendix (A2) – Phillips Curve in Taylor Model, 3 period contract lengths, closed prices

Assume that wages, once set, are unable to be changed for three periods, and an equal proportion of workers – one third – renegotiate wages each period. Hence the average wage a firm faces will be

$$w_t = \frac{1}{3} \sum_{i=0}^{2} x_{t-i} \quad (A.08)$$

since wages set two periods ago will still be in effect. We still assume prices are a constant markup over domestic wages as in A1, but now contract wages look two periods into the future,

$$x_t = \frac{\delta}{3} \sum_{i=0}^{2} w_{t+i} + \frac{(1-\delta)}{3} \sum_{i=0}^{2} p_{t+i} + \frac{\gamma}{3} \sum_{i=0}^{2} y_{t+i} + u_t \quad (A.09)$$

Combining the equations for average wages, contract wages, and prices gives the following analogue to (A.05).

$$p_t = \frac{1}{3} (x_t + x_{t-1} + x_{t-2})$$
\[ p_t = \frac{1}{9} \left( p_t + E_t p_{t+1} + E_t p_{t+2} + p_{t-1} + E_{t-1} p_t + E_{t-1} p_{t+1} + p_{t-2} + E_{t-2} p_{t-1} + E_{t-2} p_t \right) + y( y_t + E_t y_{t+1} + E_t y_{t+2} + y_{t-1} + E_{t-1} y_t + E_{t-1} y_{t+1} + y_{t-2} + E_{t-2} y_{t-1} + E_{t-2} y_t) + u_t + u_{t-1} + u_{t-2} \]

\[ = \frac{1}{9} (p_{t-2} + 2p_{t-1} + 3p_t + 2E_t p_{t+1} + E_t p_{t+2} + y \zeta^2(y_t) + \eta' ) \quad (A.10) \]

The term \( \eta' \) can be again regarded as a combination of the stochastic error term with expectational errors,

\[ \eta' = \begin{cases} u_t + u_{t-1} + u_{t-2} \\ (E_{t-1} p_t - p_t) + (E_{t-1} p_{t+1} - E_t p_{t+1}) + (E_{t-2} p_{t-1} - p_{t-1}) + (E_{t-2} p_t - p_t) \\ (E_{t-1} y_t - y_t) + (E_{t-1} y_{t+1} - E_t y_{t+1}) + (E_{t-2} y_{t-1} - y_{t-1}) + (E_{t-2} y_t - y_t) \end{cases} \quad (A.11) \]

and \( \zeta^2(y_t) = (y_{t-2} + 2y_{t-1} + 3y_t + 2E_t y_{t+1} + E_t y_{t+2}) \). Notice the symmetry of \( \zeta'(y_t) \). We can infer from this that longer contract lengths translate to prices being affected by both further lagged and expected prices and output. Hence, prices for T85 under contact lengths of 3 periods evolve according to,

\[ p_t = \frac{1}{6} (p_{t-2} + 2p_{t-1} + 2E_t p_{t+1} + E_t p_{t+2} + y \zeta^2(y_t) + \eta' ) \quad (A.12) \]

Finally, substituting the definition of inflation from Part A and rearranging terms gives,
\[ \pi_t = E_t \pi_{t+1} + \frac{1}{3} (E_t \pi_{t+2} - \pi_{t-1}) + \frac{1}{3} \zeta^2(y_t) + \frac{1}{3} \eta', \]  
(A.13)

Appendix (A3) – Phillips Curve in Taylor Model, 2 period contract lengths, relaxed purchasing power parity

Assume contract wages and average wages are set according to the same equations as in A1, namely (A.01) and (A.03). But now, prices are open to the foreign economy, e.g. \( 0 < \theta < 1 \) in

\[ p_t = \theta w_t + (1-\theta)(e_t + p_t^f) \]  
(A.14)

where an F superscript denotes a foreign variable, and \( e_t \) is the exchange rate. Let \( p_t' = e_t + p_t^f \), be the foreign price level denoted in domestic currency. Combining the equations (A.01), (A.03) and (A.14) now gives the open prices version of (A.05),

\[
\frac{1}{\theta} p_t - \frac{(1-\theta)}{\theta} (p_t') = \frac{1}{4} \begin{vmatrix}
\delta \left( \frac{1}{\theta} p_t - \frac{(1-\theta)}{\theta} (e_t + p_t^f) + \frac{1}{\theta} E_t p_{t+1} - \frac{(1-\theta)}{\theta} E_t (e_{t+1} + p_{t+1}^f) \right) \\
+ \delta \left( \frac{1}{\theta} p_{t-1} - \frac{(1-\theta)}{\theta} (e_{t-1} + p_{t-1}^f) + \frac{1}{\theta} E_{t-1} p_t - \frac{(1-\theta)}{\theta} E_{t-1} (e_t + p_t^f) \right) \\
+ (1-\delta) (p_t + E_t p_{t+1}) + (1-\delta) (p_{t-1} + E_{t-1} p_t) + \gamma (y_t + E_t y_{t+1} + y_{t-1} + E_{t-1} y_t) + u_t + u_{t-1}
\end{vmatrix}
\]
\[
\left( \frac{\delta (1 - \theta) + \theta}{\theta} \right) (2p_t + E_t p_{t+1} + p_{t-1} + (E_{t-1} - p_t)) + (1 - \theta) \left( \frac{\delta (1 - \theta)}{\theta} (2p_t' + E_t p_{t+1}' + p_{t-1}' + (E_{t-1} - p_t')) + y \zeta^{(1)}(y_t) + n_t + u_t + u_{t-1} \right)
\]

We multiply through by \( 4 \theta \) and solve the resulting parts involving \( p_t \) and \( p_t' \) individually. It is straightforward to show that domestic and foreign prices (denominated in local currency) must satisfy:

\[(4 - 2 \phi) p_t = \phi E_t p_{t+1} + \phi p_{t-1} + \phi n_t^p \] (A.16)

\[ -(1 - \theta) \left[ (4 - 2 \delta) p_t' = \delta E_t p_{t+1}' + \delta p_{t-1}' + \delta n_t'^p \right] \] (A.17)

where \( \phi = \delta (1 - \theta) + \theta \), and \( n_t \) is the expectational error for either domestic or foreign prices, \( (E_{t-1} p_t' - p_t'), i \in \{1, f' \} \). Combining these expressions with the output and stochastic shock terms gives a final expression for relaxed PPP prices in the open economy,

\[
p_t = \frac{1}{4 - 2 \phi} \left\{ \phi E_t p_{t+1} + \phi p_{t-1} + \phi n_t^p + (1 - \theta) \left( (4 - 2 \delta) p_t' - \delta E_t p_{t+1}' - \delta p_{t-1}' - \delta n_t'^p \right) + \theta (y \zeta^{(1)}(y_t) + n_t + u_t + u_{t-1}) \right\} \] (A.18)

As before, we plug in the definition of inflation, \( \pi = p_t - p_{t-1} \), to arrive at the Phillips curve for T85 under relaxed PPP and 2 period contract lengths.
\[
\pi_t = E_t \pi_{t+1} + \left[ (1-\theta)(\frac{\delta}{\phi}) \left[ E_t \pi_{t+1} - \pi_t \right] + \frac{\theta}{\phi} \left[ (4\delta - 4)(w_t - p_t) + y \zeta(y_t) \right] \right] \eta''', \quad (A.19)
\]

where \( \eta''', t = n_t - \frac{(1-\theta)\delta}{\phi} n_t^{p'} + \frac{\theta}{\phi} (n_t^y + u_t + u_{t-1}) \), a combination of foreign and domestic expectational error terms. Note that in the special case of \( \theta = 0 \), which corresponds to purchasing power parity from (A.14), the dependence on all domestic variables besides expected inflation drops out and we are left with,

\[
\pi_t = E_t \pi_{t+1} + \left[ E_t \pi_{t+1} - \pi_t \right] + \eta''', \quad (A.20)
\]

The resulting error term loses all dependence on the autoregressive wage shock in (A.03), leaving only the difference in expectational error, \( \eta''', t = n_t^p - n_t^{p'} \).

It should be noted that while it was possible to derive an explicit inflation formula under the assumption of relaxed purchasing power parity, modeling the resulting economy under modified T85 is not as straightforward. In order for equation (A.19) to agree with the other variables under modified T85, the real wage, \( w_t - p_t \), cannot appear as it is not defined. Further research is needed to find a good method for substituting for the real wage.

**Appendix A4: Finding a Simple Monetary Rule**

Suppose an objective loss function over all time periods is given by:
constrained by the Phillips-type equation:  \[ \pi_i = \beta E_t(\pi_{i+1}) + \lambda y_t + u_t, \] where all variables and coefficients are as in CCG (2003), and that a solution to the constrained optimization problem exists of the form:  \[ y_t = b_y u_t, \quad \pi_t = b_\pi u_t \] with \( b_y, b_\pi \in \mathbb{R} \) and \( \forall t \), then the optimizing coefficients can be found analytically.

Proof:

Plugging \( b_y \) and \( b_\pi \) into the constraint gives that

\[
L_t = -\frac{(1-y)}{2} \Lambda E_t \left[ \sum_{i=0}^{\infty} \beta^i \left( \frac{\lambda b_y + 1}{1 - \beta \rho} \right)^2 + \alpha b_y^2 \right].
\] (A.21)

Substituting into the constraint and simplifying, we have,

\[
L_t = -\frac{(1-y)}{2} \Lambda E_t \left[ \sum_{i=0}^{\infty} \beta^i \left( \frac{\lambda b_y + 1}{1 - \beta \rho} \right)^2 + \alpha b_y^2 \right] = -\frac{(1-y)}{2} \Lambda \left( \sum_{i=0}^{\infty} (\rho^2 \beta)^i \right) \left( \frac{\lambda b_y + 1}{1 - \beta \rho} \right)^2 + \alpha b_y^2 u_t^2, \] (A.22)

since \( u_t \) is defined as an AR(1) process with coefficient \( \rho \). Both the parameters \( \rho \) and \( \beta \) are less than 1, hence \( \sum_{i=0}^{\infty} (\rho^2 \beta)^i \) is a convergent sum. The variable \( u_t \) is known at time t. Thus first order conditions on \( b_y \) give us:
\[ \lambda^2 b_y + \lambda + (1 - \beta \rho)^2 \alpha b_y = 0 \Rightarrow b_y = \frac{-\lambda}{\lambda^2 + (1 - \beta \rho)^2 \alpha} = \frac{-\xi}{\lambda \xi + (1 - \beta \rho)^2} \]
\[ \Rightarrow b_n = \frac{(1 - \beta \rho)}{(\lambda \xi + (1 - \beta \rho)^2)} \]

Appendix A5: Modifying T85 to allow for policy rules depending on inflation

In order to modify T85 to allow for inflationary policy rules of the form: 
\[ i_t = \alpha E_t[\pi_t + 1] \]
we must
(1) replace equations (T851.B) – (T851.D) with one of the inflation equations derived in Appendix A1,2, or 3. It is also important to remove the price level from equation (T851.A). This is done by solving forward and summing:

\[ y_t = -dr_t + f(e_t + p_t' - p_t) + gy'_t \quad + \quad y_{t-1} = -dr_{t-1} + f(e_{t-1} + p_{t-1}' - p_{t-1}) + gy'_{t-1} = \]
\[ y_t = y_{t-1} - d(r_t - r_{t-1}) + f(e_t - e_{t-1} + \pi' - \pi) + g(y'_t - y'_{t-1}) \]

Appendix A6: Dynare Code to Implement CCG

// Clarida, Gali and Gertler (2002) model
// edited for numeric solvability
// Characteristics:
// complete markets, open economy, intermediate goods, nom rigid through firms, random wage & prod shocks, non-coop monetary policy

var y_squig, y, nom_int, infl, real_int, labor_shock,
    y_squig2, y2, nom_int2, infl2, real_int2, labor_shock2,
    y_bar, y_bar2, mc, mc2, tech_shock, tech_shock2, y_sqsq, y_sqsq2; // e,p,p2,r,r2
varexo lerr, lerr2, terr, terr2;

// %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
// % define parameters here  
// %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  

parameters sig_0, sig, kap_0, sig_02, kap_02, lambda, theta, beta, lambda2, kap, kap2, gamma, phi, zed, zed2, rho, rho2,  
delta, epsi, epsi2, tao, tao2, alpha, alpha2,a,a2,b,b2,c,c2,d,d2,e,e2,f,f2,b_y,b_pi,b_y2,b_pi2,g,g2,big_phi;

// degree of consumption preference
sig = .2;

// degree leisure disutility
phi = 1;

// foreign country mass (relative to home)
gamma = .5;

// household's utility discount parameter
beta = .99;

// prob that firm keeps price fixed each period
theta = .9;

// price elast of demand (home & foreign)
zed = 1;
zed2 = 1;

// shock persistence (home & foreign)
rho = .95;
rho2 = .95;
epsi = .95;
epsi2 = .95;

// everything else
kap = sig*(1-gamma) + gamma + phi;
kap2 = (sig*gamma) + (1-gamma) + phi;
kap_0 = (sig*gamma) - gamma;
kap_02 = (sig-1)*(1-gamma);
sig_0 = sig - kap_0;
sig_02 = sig - kap_02;
lambda = ((1-theta)*(1-(beta*theta))/theta)*kap;
lambda2 = ((1-theta)*(1-(beta*theta))/theta)*kap2;
delta = (1-theta)*(1-(beta*theta))/theta;
tao = .8;
tao2 = .8;
// M-Policy Coeffs
alpha = .9;
alpha2 = .9;
a = (1 + ((zed*sig_0*(1-rho))/rho)); // coeff on i = pi(1) + rr
a2 = (1 + ((zed2*sig_02*(1-rho2))/rho2));
b = (kap_0 / kap)*(a-1); // coeff on i = pi(1) + pi2(1) + rr in coordination
b2 = (kap_02 / kap2)*(a2-1);
c = (rho + sig_0*zed*(1-rho)); // coeff on i = pi + rr, non-coord
C2 = (rho2 + sig_02*zed2*(1-rho2));
d = c / ((1-beta*rho) + lambda*zed); // coeff on i = u + rr, non-coord
d2 = c2 / ((1-beta*rho2) + lambda2*zed2);
e = ((rho/zed) + sig_0*(1-rho)); // coeff on i = y + rr, nc
e2 = ((rho2/zed2) + sig_02*(1-rho2));

b_y = (-1*lambda)/(lambda^2 + (lambda/zed)*(1-beta*rho)^2);
b_p = (lambda*b_y + 1) / (1-beta*rho);
f = ((b_y/b_p)*((rho-1)/rho)*sig_0 + rho);

b_y2 = (-1*lambda2)/(lambda2^2 + (lambda2/zed2)*(1-beta*rho2)^2);
b_p2 = (lambda2*b_y2 + 1) / (1-beta*rho2);
f2 = ((b_y2/b_p2)*((rho2-1)/rho2)*sig_02 + rho2);

g = ((b_y/b_p)*((rho-1)/rho)*sig_0 + 1);
g2 = ((b_y2/b_p2)*((rho2-1)/rho2)*sig_02 + 1);

big_phi = (delta*(1-sig)*gamma*(1-gamma)) / zed;
model(linear);

////////////////////////////////////////////////////////////////////////////////////////
// Country 1
////////////////////////////////////////////////////////////////////////////////////////

// Y squig definition
y = y_squig + y_bar;

// IS Curve
y_squig = y_squig(1) - ((sig_0)^(-1))*(nom_int - infl(1) - real_int);

///////////
// Splitting up Phillips Curve to get more interaction
// Marginal Cost Function (11 + 20 + 30 + 33)
mc = labor_shock + kap*y + kap_0*y2 - (1+phi)*tech_shock;

// Inflation Rule
infl = delta*mc + beta*infl(1);

////////
// Real Interest Rate
//real_int = sig_0*(y_bar(1) - y_bar) + kap_0*(y2(1) - y2);  
real_int = sig_0*(y_bar(1) - y_bar) + kap_0*(y_bar2(1) - y_bar2);
//real_int = nom_int - infl;
//real_int = 0;

// Flexible Prices Output Level
y_bar = (1/kap)*((1+phi)*tech_shock - kap_0*y2);
//y_bar = 0;

// World Flexible Prices Output Level
y_sqsq = y_squig - (kap_0 / kap)*(y_sqsq2);

// Labor Shock
labor_shock = rho*labor_shock(-1) + lerr;

// Technology Shock
tech_shock = epsi*tech_shock(-1) + terr;

///////////////////////////////////////////////////////////////////////////
// Country 2
///////////////////////////////////////////////////////////////////////////

// Y squig definition
y2 = y_squig2 + y_bar2;

// IS Curve
y_squig2 = y_squig2(1) - ((sig_02)^(-1))*(nom_int2 - infl2(1) - real_int2);

//////////////
// Splitting up Phillips Curve to get more interaction
// Marginal Cost Function
mc2 = labor_shock2 + kap2*y2 + kap_02*y - (1+phi)*tech_shock2;

// Inflation Rule
infl2 = delta*mc2 + beta*infl2(1);

///////////
// Real Interest Rate
//real_int2 = sig_02*(y_bar2(1) - y_bar2) + kap_02*(y(1) - y);
real_int2 = sig_02*(y_bar2(1) - y_bar2) + kap_02*(y_bar(1) - y_bar);
//real_int2 = nom_int2 - infl2;
//real_int2 = 0;

// Flexible Prices Output Level
y_bar2 = (1/kap2)*((1+phi)*tech_shock2 - kap_02*y);
//y_bar2 = 0;

// World Flexible Prices Output Level
y_sqsq2 = y_squig2 - (kap_02 / kap2)*(y_sqsq);

// Labor Shock
labor_shock2 = rho2*labor_shock2(-1) + lerr2;
// Technology Shock

tech_shock2 = epsi2*tech_shock2(-1) + terr2;


// Country 1
//nom_int = real_int + d*labor_shock;
//nom_int = real_int + c*infl;
//nom_int = real_int + (c-rho)*infl - (1/zed)*y_squig(1);
//nom_int = real_int - (1/zed)*a*y_squig(1);
//nom_int = real_int - (1/zed)*c*y_squig;
nom_int = 1.1*infl(1);
//nom_int = a*infl(1) + b*infl2(1);

//nom_int = real_int + a*infl(1) + b*infl2(1);
//nom_int = real_int + a*infl(1);
//nom_int = real_int + f*infl;
//nom_int = real_int + g*infl(1);

// Country 2
//nom_int2 = real_int2 + d2*labor_shock2;
//nom_int2 = real_int2 + c2*infl2;
//nom_int2 = real_int2 + (c2-rho2)*infl2 - (1/zed2)*y_squig2(1);
//nom_int = real_int - (1/zed)*a*y_squig(1);
//nom_int = real_int - (1/zed)*c*y_squig;
nom_int2 = 1.1*infl2(1);
//nom_int2 = a2*infl2(1) + b2*infl(1);

//nom_int2 = real_int2 + a2*infl2(1) + b2*infl(1);
//nom_int2 = real_int2 + a2*infl2(1);
//nom_int2 = real_int2 + f2*infl2;
//nom_int2 = real_int2 + g2*infl2(1);
// Definition of Price Level

//p = e + p2;
//infl = p - p(-1);
//infl2 = p2 - p2(-1);
//r = nom_int - infl;
//r2 = nom_int2 - infl2;
end;

// Steady state values
initval;

// Steady state for country 1

y_squig = 0;
y = 0;
y_bar = 0;
nom_int = 0;
infl = 0;
real_int = 0;
labor_shock = 0;
mc = 0;

// Steady state for country 2

y_squig2 = 0;
y2 = 0;
y_bar2 = 0;
nom_int2 = 0;

infl2 = 0;

real_int2 = 0;

labor_shock2 = 0;

mc2 = 0;

end;

steady;

shocks;
var lerr;
//stderr 0;
periods 1;
values .01;
var lerr2;
stderr 0;
//periods 1;
//values .01;
var terr;
stderr 0;
//periods 1;
//values .01;
var terr2; stderr 0;
end;

cHECK;
simul(periods=100);
//stoch_simul(noprint,linear,irf=0,periods=100);

//stoch_simul(order=1,nocorr,nomoments,noprint,irf=0);
Appendix A7: Dynare Code to Implement Canonical T85

// Taylor (1985) Model w/ M-Policy
// Characteristics:
// Staggered Wages, perfect capital mobility, 2 countries, rational expectations,
// monetary policy (aug taylor rule), cost-push (nom) shock

var x, w, p, y, i, e,
    x2, w2, p2, y2, i2, pi, pi2, u, u2, r, r2, m, m2;

varexo err, err2, supply_err, supply_err2;

parameters delta, gamma, theta, d, f, g, b, a,
    delta2, gamma2, theta2, d2, f2, g2, b2, a2, s_coef, s_coef2
    alpha, beta, alpha2, beta2, kap, kap2, pf, pf2;

delta = 0.5;
delta2 = 0.5;

gamma = 1;
gamma2 = 1;

theta = 0.8;
theta2 = 0.8;

d = 1.2;
d2 = 1.2;

f = 0.1;
f2 = 0.1;

g = 0.1;
g2 = 0.1;

b = 4.0;
b2 = 4.0;

a = .2;
a2 = .2;

s_coef = .95;
s_coef2 = .95;

alpha = 1.2;
alpha2 = 1.2;

beta = 1;
beta2 = 1;

kap = 0;
kap2 = 0;

pf = 1.4542;
pf2 = 1.4542;

model(linear);

///////////////////////////////////////////////////////////////////////////
// Country 1
///////////////////////////////////////////////////////////////////////////

// Contact Wages
x = delta*(w + w(1) + w(2))*(1/3) + (1-delta)*(p + p(1) + p(2))*(1/3) + gamma*(y + y(1) + y(2))*(1/3) + u;
//x = delta*(w + w(1))*(1/2) + (1-delta)*(p + p(1))*(1/2) + gamma*(y + y(1))*(1/2) + u;

// Nominal Wage
w = (1/3)*(x + x(-1) + x(-2));
//w = (1/2)*(x + x(-1));
// Price Level
p = theta*w + (1-theta)*(e + p2);

// Output (IS Curve)
y = (-d)*r + f*(e + p2 - p) + g*y2;

// LM Curve
//m = p + (-b)*i + a*y;

// M-Policy Rule (either LM or MPolicy can be switched on)
i = alpha*p;
//i = alpha*pi(1) + beta*r;
//r = alpha*p;
//i = pf*pi;

// Real Int Rate
r = i - pi(1);
//r = (u(1) - u) + alpha*(y(1) - y);

// Expected Inflation
pi = p - p(-1);

// Money Supply (exog)
m = err;

// Supply Shock (exog)
u = s_coef*u(-1) + supply_err;

///////////////////////////////////////////////////////////////////////////
// Country 2
///////////////////////////////////////////////////////////////////////////

// Contact Wages
x2 = delta2*(w2 + w2(1) + w2(2))*(1/3) + (1-delta2)*(p2 + p2(1) + p2(2))*(1/3) + gamma2*(y2 + y2(1) + y2(2))*(1/3) + u2;
//x2 = delta2*(w2 + w2(1))*(1/2) + (1-delta2)*(p2 + p2(1))*(1/2) + gamma2*(y2 + y2(1))*(1/2) + u2;
// Nominal Wage
w2 = (1/3)*(x2 + x2(-1) + x2(-2));
//w2 = (1/2)*(x2 + x2(-1));

// Price Level
p2 = theta2*w2 + (1-theta2)*(p - e);

// Output (IS Curve)
y2 = (-d2)*r2 - f2*(e + p2 - p) + g2*y;

// LM Curve
// m2 = p2 + (-b2)*i2 + a2*y2;

// M-Policy Rule (either LM or MPolicy can be switched on)
i2 = alpha2*p2;
//i2 = alpha2*pi2 + beta2*r2;
//r2 = alpha2*p2;
//i2 = pf2*pi2;

// Real Int Rate
r2 = i2 - pi2(1);
//r2 = (u2(1) - u2) + alpha2*(y2(1) - y2);

// Expected Inflation
pi2 = p2 - p2(-1);

// Money Supply (exog)
m2 = err2;

// Supply Shock (exog)
u2 = s_coef*u2(-1) + supply_err2;

/////////////////////////////////////////////////////////////////////
// Other
/////////////////////////////////////////////////////////////////////

// Capital Mobility eq
\[
i = i_2 + e(+1) - e;
\]

\[
//p = e + p_2;
\]

end;

// Steady state values

initial;

// Steady state for country 1

x = 0;
w = 0;
p = 0;
y = 0;
r = 0;
i = 0;
pi = 0;
m = 0;
e = 0;
u = 0;

// Steady state for country 2

x_2 = 0;
w_2 = 0;
p_2 = 0;
y_2 = 0;
r_2 = 0;
i_2 = 0;
pi_2 = 0;
m2 = 0;
u2 = 0;

end;

steady;

shocks;
var err; stderr 0;
//periods 1:999;
//values 1;
var err2; stderr 0;
//var pi;
//stderr 0;
//var pi2;
//stderr 0;
var supply_err;
//stderr 0.01;
periods 1;
values .01;
var supply_err2; stderr 0;
end;

check;

simul(periods=100);
//stoch_simul(linear,irf=0);

//stoch_simul(order=1,nocorr,nomoments,noprint,irf=0);

**Appendix A8: Dynare Code to Implement Modified T85**

Note: I only publish the country 1 % 2 blocks as the rest of the code is identical to the canonical model.
// Taylor (1985) Model w/ M-Policy – Modified for inflation based policy rules
// Characteristics:
// Staggered Wages, perfect capital mobility, 2 countries, rational expectations,
// monetary policy (aug taylor rule), cost-push (nom) shock

///////////////////////////////////////////////////////////////////////////
// Country 1
///////////////////////////////////////////////////////////////////////////

// Output (IS Curve)
y = y(-1) - d*(r - r(-1)) + f*(e - e(-1) + pi2 - pi) + g*(y2 – y2(-1));

// LM Curve
//m = p + (-b)*i + a*y;

// Real Int Rate
r = i - pi(1);
//r = (u(1) - u) + alpha*(y(1) - y);

// Expected Inflation
// 2 Crtct Closed Prices
pi = pi(1) + gamma*(y(-1) + 2*y + y(1)) + u;
// 2 Crtct Open Prices (PPP)
//pi = pi(1) + ((1-theta)*(delta/phi)*(e(1) - 2*e + e(-1) + pi2(1) - pi2)) + (theta/phi)*gamma*(y(-1) + 2*y + y(1)) + u;
// 3 Crtct Closed Prices
//pi = pi(1) + (1/3)*(pi2 - pi(-1)) + (gamma/3)*(y(-2) + 2*y(-1) + 3*y + 2*y(1) + y(2)) + (1/3)*u;

// Money Supply (exog)
//m = err;

// Supply Shock (exog)
\[ u = s_{\text{coef}}u(-1) + \text{supply\_err}; \]

///////////////////////////////////////////////////////////////////////////
// Country 2
///////////////////////////////////////////////////////////////////////////

// Output (IS Curve)
\[ y_2 = y_2(-1) - d_2(r_2 - r_2(-1)) - f_2(e - e(-1) + \pi_2 - \pi) + g_2(y - y(-1)); \]

// LM Curve
\[ m_2 = p_2 + (-b_2)i_2 + a_2y_2; \]

// Real Int Rate
\[ r_2 = i_2 - \pi_2(1); \]
\[ r_2 = (u_2(1) - u_2) + \alpha_2(y_2(1) - y_2); \]

// Expected Inflation
// 2 Crtct Closed Prices
\[ \pi_2 = \pi_2(1) + \gamma_2(y_2(-1) + 2y_2 + y_2(1)) + u_2; \]

// 2 Crtct Open Prices (PPP)
\[ \pi_2 = \pi_2(1) - ((1-\theta_2)(\delta_2/\phi_2)(e(1) - 2e + e(-1) + \pi(1) - \pi)) + (\theta_2/\phi_2)\gamma_2(y_2(-1) + 2y_2 + y_2(1)) + u_2; \]

// 3 Crtct Closed Prices
\[ \pi_2 = \pi_2(1) + (1/3)(\pi_2(2) - \pi_2(-1)) + (\gamma_2/3)(y_2(-2) + 2y_2(-1) + 3y_2 + 2y_2(1) + y_2(2)) + (1/3)u_2; \]
// Money Supply (exog)
//m2 = err2;

// Supply Shock (exog)
u2 = s_coef*u2(-1) + supply_err2;

///////////////////////////////////////////////////////////////////////////

// Other

 ///////////////////////////////////////////////////////////////////////

// Capital Mobility eq
i = i2 + e(1) - e;
//p = e + p2;


///////////////////////////////////////////////////////////////////////////

// M-Policy Rule (either LM or MPolicy can be switched on)

///////////////////////////////////////////////////////////////////////////

/////Country 1
i = alpha*pi;

//i = pa*pi(1) + pb*pi2(1);
//i = pa*pi(1);
//i = pf*pi;
//i = pg*pi(1);

/////Country 2
i2 = alpha2*pi2;
// i2 = pa2*pi2(1) + pb2*pi(1);
// i2 = pa2*pi2(1);
// i2 = pf2*pi2;
// i2 = pg2*pi2(1);
end;