Outline

1. Calculus
   1.1 Overview
   1.2 Geometric Interpretation
   1.3 Ex. Area under a line
2. Derivatives
3. Connection to Histograms and Cumulative Density
4. Logs & Exponents
Georg Friedrich Bernhard Riemann (1826-66)
The Fundamental Theorem of Calculus

\[ \int_{a}^{b} f(x) \, dx = F(b) - F(a) \]
Integral Rules

There are lots of “rules” for specific integrals.

- **Powers:** \( f(x) = x^a \), then \( F(x) = \frac{1}{a+1} x^{a+1} \).  
  Ex. Integral of \( x \) is \( \frac{1}{2} x^2 \).

- **Exponential:** \( f(x) = e^x \), then \( F(x) = e^x \).

- **Sin/Cos:** \( f(x) = \cos(x) \), then \( F(x) = \sin(x) \).

- ... 

We’ll be mostly interested in geometric problems.
Example 1. Area under a line.

Definition

A line is an equation of the form $y = mx + b$. 

$m$ is the slope and $b$ is the intercept.

$$m = \frac{y_1 - b}{x_1 - 0}$$
Example 1. Area under a line.

What is the area in between 1 and 3 of the line passing through points \((-1, 5)\) and \((6, 1)\)?

1. Get equation for line \( m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 5}{6 - (-1)} = \frac{-4}{7} \)

   \( b = y - mx = 5 - \left( \frac{-4}{7} \right) (-1) = 5 - \frac{4}{7} = 4 \frac{3}{7} \)

2. Plot and fill area
Example 1. Area under a line.

What is the area in between 1 and 3 of the line passing through points \((-1, 5)\) and \((6, 1)\)?

3. Calculate

\[
\int_{1}^{3} \left( \frac{-4}{7}x + 4\frac{3}{7} \right) \, dx = \{triangle\} + \{square\} \\
= \frac{1}{2}(2)(1\frac{5}{7}) + (2)(2\frac{5}{7}) \\
= 1\frac{5}{7} + 5\frac{3}{7} = 7\frac{1}{7}
\]
Derivatives

- Many applications & uses
  1. Slope of a line
  2. Instantaneous velocity
  3. Density of a distribution \( f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2} \)

- Think of as opposite of integral

If \( F(x) = \int_a^x f(y) dy \), then \( dF(x) = f(x) \).

- Usually denoted as \( dF(x) \), or \( F'(x) \).
Define: $F(x) = \{\% \text{ of people who smoke } \leq x\}$
Connection between Integrals, Derivatives and “% per” in Histograms

So $F(10) = 15\%$

And $F(30) = 65\%$
Connection between Integrals, Derivatives and “% per” in Histograms

In general*,

\[ F(x) = \int_0^x f(y) \, dy \]

where \( f(y) \) is the height of the histogram.

Probability density function (pdf) \hspace{2cm} Cumulative density function (cdf)
Logs & Exponents

An exponent: $2^x$, $10^x$, $e^x$ (exponential)

A log: $\log_2(y)$, $\log_{10}(y)$, $\ln(y)$ (natural log)
  - “the number $x$ so that $2^x = y$”
  - $\log_a(a^x) = x$ and $a^{\log_a(x)} = x$
  - We (statisticians) sometimes take logs of data if it is heavily skewed.

A couple rules:
  - $\log(xy) = \log(x) + \log(y)$, $\log(x/y) = \log(x) - \log(y)$
  - $e^x e^y = e^{x+y}$, $e^x / e^y = e^{x-y}$
Logs & Exponents

source: http://my.execpc.com/~dluisa/ArithVsLog.html
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