1. Suppose we observe an $n$ dimensional Gaussian vector $y \sim \mathcal{N}(\mu, I_n)$ and wish to estimate a linear transformation $L\mu$ of the mean vector $\mu$ by means of a statistic $T(y)$. Under appropriate conditions which you may want to discuss, give an unbiased estimate for the MSE, namely, $E \|T(y) - L\mu\|^2$ of this estimate.

2. FDR thresholding. Suppose we wish to estimate $\mu$ from $y \sim \mathcal{N}(\mu, I)$. Compare the relative merits of FDR and ‘universal’ thresholding through a simulations study. By universal thresholding, I mean that the value of the threshold is set at $\sqrt{2\log p}$ or some finite sample version of this (Bonferroni level).

3. Suppose I have a linear model in which each row of $X$ is i.i.d. $\mathcal{N}(0, I_p)$ and that we fit a linear model

$$y = X\beta + z, \quad z \sim \mathcal{N}(0, \sigma^2 I_n)$$

via $C_p$; that is, we solve

$$\min_{\hat{\beta}} \|y - X\hat{\beta}\|^2 + 2\sigma^2\|\hat{\beta}\|_0.$$ 

Imagine $p$ is fixed and large. In the limit where the sample size $n$ goes to infinity, how many false discoveries do you expect to make? A false discovery is a variable included in the $C_p$ model which does not belong (to the true model). You may make any assumption you like about $\beta$. E.g. you can assume $\beta = 0$ if this makes life easier.