Solution to the dowry problem

Since we do not know anything about the distribution of the dowries, it seems that a reasonable strategy is to wait until a certain number \( k \) of daughters have been presented, and then pick the highest dowry thereafter.

Let \( D \) be the position of the highest dowry, e.g. \( D = 1 \) if the first lady is that with the highest dowry, \( D = 2 \) if the first lady is that with the highest dowry. We express

\[
P(\text{Win}) = \sum_{i=1}^{100} P(\text{Win} | D = i) P(D = i).
\]

1. \( P(D = i) = 1/100. \)
2. The probability that you will choose the largest dowry if it is in the \( i \)th position is equal to the probability that the highest of the first \( i - 1 \) dowries belongs to one of the first \( k \) ladies. This equals \( k/(i - 1) \). If the highest dowry of the first \( i - 1 \) were not in the first \( k \) ladies you would choose it before getting to the largest dowry, thus losing the game.

The probability of winning is then given by

\[
\pi_k = \frac{1}{100} \sum_{i=k+1}^{100} \frac{k}{i-1}
\]

Now you want to choose \( k \) such that this probability is maximum. Numerical calculations show that \( k = 37 \) as suggested by the plot below. The probability of winning is about 37.10%.

Let \( n \) be the number of dowries. Note that

\[
1 + 1/2 + 1/3 + \ldots + 1/m \sim \log(m)
\]

and therefore

\[
\pi_k \approx \frac{k}{n} (\log(n) - \log(k)) = -\frac{k}{n} \log(k/n)
\]

Hence, we want to maximize \(-x \log x\) and the maximum is at \( 1/e \) for which the value is \( 1/e \). In general the answer is going to pass on \( n/e \) ladies, where \( n \) is the total number of ladies. Because the answer must be an exact integer, check the few integers just above the optimal value.