Homework 2
Due Thursday, October 10 (in class)

Please write neat and complete solutions to the problem sets. “Neat” means well structured, both esthetically and logically. “Complete” means that the grader will need to see a sufficient amount of explanations and details to give you full credit, even if the question only asks for a numerical answer. Thanks.

1. Laub, Chapter 2, exercise 2.
2. Laub, Chapter 2, exercise 3.
3. Laub, Chapter 2, exercise 7.
4. This problem examines nilpotent matrices.
   (a) Find a square matrix \( A \), whose entries are not all zeros, such that \( A^2 = 0 \).
   (b) Exhibit a nonzero vector that belongs to the nullspace of the matrix you just constructed.
   (c) In general, prove that if a matrix \( B \) satisfies \( B^2 = 0 \), then it cannot be invertible.
5. Do either (a) or (b) (not both):
   (a) Laub, Chapter 3, exercise 1.
   (b) Given a matrix \( A \in \mathbb{R}^{m \times n} \), write a function that returns the representation of \( A \) with respect to the bases \( V = [v_1, \ldots, v_n] \) of \( \mathbb{R}^n \) and \( W = [w_1, \ldots, w_m] \). Your function should take as input three matrices, namely, \( A \in \mathbb{R}^{m \times n} \), \( V \in \mathbb{R}^{n \times n} \) and \( W \in \mathbb{R}^{m \times m} \) (the \( j \)th column of \( V \) is the vector of coordinates of \( v_j \in \mathbb{R}^n \) and, similarly, the \( j \)th column of \( W \) is the vector of coordinates of \( w_j \in \mathbb{R}^m \)). [Hint: you will probably need a matrix inverse which I am not asking you to code. You can just make a library, e.g. in Python you might want to call \texttt{np.linalg.inv}.] Use your function to give the answer to (a)!