Homework 2

Due date: Wednesday, October 18

1. Threfethen and Bau, Chapter 21, Problem 21.2.

2. Threfethen and Bau, Chapter 21, Problem 21.6.

3. Threfethen and Bau, Chapter 23, Problem 23.3.

4. Threfethen and Bau, Chapter 12, Problem 12.2.

5. Suppose we know how to solve $Ax = b$ easily (i.e. we have a ‘fast’ algorithm). We now want to solve a system $Hx = b$ where $H = A + uv^T$ with $u$ and $v$ two vectors of length $n$; that is, $H$ is equal to $A$ up to an additive matrix of rank 1.

(a) Suppose $A = I$. Show that the inverse of $I + uv^T$ equals $I + \sigma uv^T$, where $\sigma$ is a scalar you will calculate.

(b) Using the result of the previous section, show that the inverse of $A + uv^T$ is given by $A^{-1} + \sigma_A(A^{-1}uv^TA^{-1})$, where $\sigma_A$ is a scalar you will calculate.

(c) If you have a fast solver for $Ax = b$, show how to build a fast solver for $Hx = b$ where $H = A + uv^T$.

(d) Bonus: Write a matlab routine that solves $Hx = b$ where $H$ is as above and $A$ is an orthonormal matrix (with orthonormal rows and columns). Your function should take as input $A$, $u$, $v$ and $b$ and return the solution $x$. 