310c: Homework Sets 2019

Homework rules: Arguments should be precise, concise and correct. We expect rigorous and complete proofs. Answers should be clean and legible. Please use a separate sheet of paper for each problem (easier for you to work with, and easier to grade), and staple together all the sheets you hand in. Submit solutions only for 4 of the 5 assigned exercises in each problem set (only first 4 exercises submitted in each set will be graded). When solving one exercise you may cite any other exercise, without proving it. Problems can be solved in a group effort, but students should independently write their own solutions!

Homework problems marked by a * are likely to be more difficult.

Homework 0: due April 4th.
Review the setting and construction of Kolmogorov’s extension theorem (in Section 1.4.2), as well as Definition 3.4.8 of the Poisson process and the definition and main properties of Gaussian random vectors (from Section 3.5).

Homework 1: due April 11th.
Review Definitions 5.1.1-5.1.4, 5.1.11 and 5.1.34 of filtration, martingale, stopping times and their σ-algebras, in the discrete time setting.
Exercises 7.1.11[only (c)(d)], 7.1.12*, 7.2.13 and 7.2.18 (solution provided also for Exercises 7.2.3, 7.2.7 and 7.2.9* which are not part of homework).

Homework 2: due April 18th.
Read everything in Chapter 7 that was not covered in lecture by Apr. 14th. In addition, review Definition 5.5.17 and Lévy’s downward theorem on the convergence of RMG-s.
Exercises 7.3.11, 7.3.10, 7.3.16 and 8.1.10 (solution provided also for Exercises 7.3.9 and 7.3.17 which are not part of homework).

Homework 3: due April 25th.
Review Definitions 6.1.1 and 6.1.2, the construction of Markov chains via Propositions 6.1.4-6.1.8 and the strong Markov property as in Proposition 6.1.16.
Exercises 8.1.11, 8.2.6, 8.2.7, 8.2.10 and 8.2.21 (solution provided also for Exercise 8.1.12 which is not part of homework).

Homework 4: due May 2nd.
Review the theory of weak convergence in topological spaces, in particular Definitions 3.2.17, 3.2.20 and 3.2.31, Prohorov’s theorem 3.2.34 and the Portmanteau theorem 3.5.2. Then, read everything in Sections 8.1-8.3.1 that was not covered in lectures.
Exercises 8.2.30, 8.2.33, 8.2.35, 8.2.36 and 8.3.4 (solution provided also for Exercises 8.2.32 and 8.2.38 which are not part of homework).

Homework 5: due May 9th.
Before the first lecture on Section 9.2, read Exercises 3.5.18 and 5.2.10 about the random walk, which we use without a proof and Exercise 3.3.34 (about stable limit laws). Later, review the material about Glivenko-Cantelli (Theorem 2.3.6) and order statistics for the uniform measure (Exercise 3.4.11). In addition, read everything in Sections 8.3.2 and 9.1 that was not covered in lectures.
Exercises 8.3.9, 8.3.10, 8.3.20, 8.3.22 and 9.1.1 (solution provided also for Exercises 8.3.17 and 9.1.7 which are not part of homework).
Homework 6: due May 16th.
Read everything in Section 9.2 that was not covered in lectures by May 12th.

Exercises 9.1.8, 9.1.11, 9.1.14, 9.1.16 and 9.1.19 (solution provided also for Exercises 9.1.16 and 9.1.18 which are not part of homework).

Homework 7: due May 21st.
Read everything in Subsections 9.2.1 and 9.2.2 which was not covered in lectures by May 16th.

Exercises 9.2.8, 9.2.10, 9.2.14, 9.2.18 and 9.2.25 (solution provided also for Exercises 9.2.13, 9.2.6 and 9.2.12 which are not part of homework).