1. **HPS 3.11**
   Let $X(t)$ be the number of customers in service at time $t$ in the infinite server queue ($M/M/\infty$ of T&K IX.2.2 or Ross pg. 234). Compute the mean and the variance of $X(t)$ in case $X(0) = x$.

2. **HPS 3.15**
   Let $X(t) = X_1(t) + X_2(t)$ be the number of customers in service by the infinite server queue at time $t$, where $X_1(t)$ is the contribution of customers arriving in the time interval $(0, t]$ while $X_2(t)$ is the contribution of the customers present at time 0. Suppose that the initial distribution $\pi_0$ of $X(0)$ is a Poisson distribution of parameter $\nu$.
   (a) Use the formula $P(X_2(t) = k) = \sum_{x=k}^{\infty} \pi_0(x) P_x(X_2(t) = k)$, to show that $X_2(t)$ has the Poisson distribution with parameter $\nu e^{-\mu t}$.
   (b) Use the result of (a) to show that $X(t)$ then has a Poisson distribution with parameter $\xi + (\nu - \xi)e^{-\mu t}$, for $\xi = \lambda/\mu$.
   (c) Conclude that $X(t)$ has the same distribution as $X(0)$ if and only if $\nu = \lambda/\mu$.

3. **HPS 3.14**
   Suppose $d$ particles are distributed into two boxes. A particle in box 0 remains in that box for a random length of time that is exponentially distributed with parameter $\lambda$ before going to box 1. A particle in box 1 remains in that box for a random length of time that is exponentially distributed with parameter $\mu$ before going to box 0. The particles act independently of each other. Let $X(t)$ denote the number of particles in box 1 at time $t \geq 0$. Then, $X(t)$ is a birth and death process on $\{0, \ldots, d\}$.
   (a) Find the birth and death rates.
   (b) Find $P_{x\rightarrow y}(t)$. Hint: $X(t) = X_0(t) + X_1(t)$, where $X_i(t)$ is the number of particles in box 1 at time $t$ that started at box $i$ at time 0. Given $X(0) = x$, then $X_0(t)$ and $X_1(t)$ are two independent binomial variables with parameters defined in terms of $x$ and the transition function of the two-state birth and death process.
   (c) Find $E_x(X(t))$. 
4. **Ross, 2.30, see his Section 2.4**
Let \( T_1, T_2, \ldots \) denote the interarrival times of events of a nonhomogeneous Poisson process having intensity function \( \lambda(t) \).

(a) Are the \( T_i \) independent?
(b) Are the \( T_i \) identically distributed?
(c) Find the distribution of \( T_1 \).
(d) Find the distribution of \( T_2 \).

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5. **Taylor & Karlin, 5.4, Problem #3**
Let \( W_1, W_2, \ldots \) be the waiting times in a Poisson process \( X(t) \) of rate \( \lambda \). Under the condition that \( X(1) = 3 \), determine the joint distribution of \( U = W_1/W_2 \) and \( V = (1 - W_3)/(1 - W_2) \).

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6. **Ross, 2.33**
A two-dimensional Poisson process is a process of events in the plane such that (i) for any region of area \( A \), the number of events in \( A \) is Poisson distributed with mean \( \lambda A \), and (ii) the numbers of events in nonoverlapping regions are independent. Consider a fixed point, and let \( X \) denote the distance from that point to its nearest event, where distance is measured in the usual Euclidean manner. Show that

(a) \( P\{X > t\} = e^{-\lambda \pi t^2} \).
(b) \( E[X] = 1/(2\sqrt{\lambda}) \).
(c) Let \( R_i, i \geq 1 \) denote the distance from an arbitrary point to the \( i \)th closest event to it. Show that, with \( R_0 = 0, \pi R_i^2 - \pi R_{i-1}^2, i \geq 1 \), are independent exponential random variables, each with rate \( \lambda \).