1. **Taylor & Karlin, Prob. III.5.4**
   Martha has a fair die with the usual six sides. She throws the die and records the number. She throws the die again and adds the second number to the first. She repeats this until the cumulative sum of all the tosses first exceeds 10. What is the probability that she stops at a cumulative sum of 13?

2. **Absorption, read first Taylor & Karlin, III.7**
   Consider the Markov chain whose transition probability matrix is given by

   $$
P = \begin{pmatrix}
   0 & 1 & 2 & 3 \\
   0 & 1 & 0 & 0 & 0 \\
   1 & 0 & 0.2 & 0.1 & 0.3 & 0.4 \\
   2 & 0.3 & 0.2 & 0.4 & 0.1 \\
   3 & 0 & 0 & 0 & 1 \\
   \end{pmatrix}
   $$

   The transition probability matrix corresponding to the nonabsorbing states is

   $$
   Q = \begin{pmatrix}
   1 & 2 \\
   0 & 0.1 & 0.3 \\
   1 & 0.2 & 0.4 \\
   \end{pmatrix}
   $$

   Calculate the matrix inverse to $I - Q$, and from this determine
   
   (a) The probability of absorption into state 0 starting from state 1;
   (b) The mean time spent in each of states 1 and 2 prior to absorption.

3. **Transient and Recurrent states, Hoel, Port & Stone, Exercise 1.19**
   Consider the Markov chain whose transition probability matrix is given by

   $$
P = \begin{pmatrix}
   0 & 1 & 2 & 3 & 4 & 5 & 6 \\
   0 & 0.5 & 0 & 0.125 & 0.25 & 0.125 & 0 \\
   1 & 0 & 0 & 0 & 1 & 0 & 0 \\
   2 & 0 & 0 & 0 & 1 & 0 & 0 \\
   3 & 0 & 1 & 0 & 0 & 0 & 0 \\
   4 & 0 & 0 & 0 & 0 & 0.5 & 0.5 \\
   5 & 0 & 0 & 0 & 0 & 0.5 & 0.5 \\
   6 & 0 & 0 & 0 & 0 & 0.5 & 0.5 \\
   \end{pmatrix}
   $$

   (a) Determine which states are transient and which states are recurrent.
   (b) Find $\rho_0 y = P_0(T_y < \infty)$, for $y = 0, 1, \ldots, 6$. 

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**Stat 217, Problem Set #3**
Due Tuesday, January 30
4. Birth-Death chains, Hoel, Port & Stone, Exercise 1.29
Consider a birth and death chain on the nonnegative integers such that \( p_x = 1/2 \) and
\[ q_x = \frac{x^2}{2(x+1)^2} \] for \( x \geq 0 \).

(a) Show that this chain is transient.
(b) Find \( \rho_x = P_x(T_0 < \infty) \) for \( x \geq 1 \) (reading Exercise 1.26 might help here).

5. Chessboard (read Ross, Sec. 4.7)

(a) Find the stationary distribution for a random walk of a king (each of 8 adjacent squares equally likely) on an 8X8 chessboard.
(b) What about a rook (which is equally likely to move to any other square in either the same row or the same column)?

6. Taylor & Karlin, Prob. IV.1.1
Five balls are distributed between two urns, labeled A and B. Each period, an urn is selected at random, and if it is not empty, a ball from that urn is removed and placed into the other urn. In the long run, what fraction of time is urn A empty?

7. Taylor & Karlin, Prob. IV.1.13
A Markov chain has the transition probability matrix
\[
P = \begin{bmatrix}
0 & 0.4 & 0.2 \\
1 & 0.4 & 0.2 \\
2 & 0.4 & 0.4 \\
\end{bmatrix}.
\]

After a long period of time, you observe the chain and see that it is in state 1. What is the conditional probability that the previous state was state 2? That is, find
\[
\lim_{n \to \infty} Pr\{X_{n-1} = 2 | X_n = 1\}.
\]