1. **F inverse (Ross, 1.15)**
   Let $F$ be a continuous distribution function and let $U$ be a uniform $(0, 1)$ random variable.
   
   (a) If $X = F^{-1}(U)$, show that $X$ has distribution function $F$.
   (b) Show that $-\log(U)$ is an exponential random variable with mean 1.
   (c) What is the distribution of $-\log U^3$?

2. **Poisson (TK Problems I.3.7, II.4.2; Ross, 1.8)**
   Let $X_1$ and $X_2$ be independent Poisson random variables with means $\lambda_1$ and $\lambda_2$.
   
   (a) Find the distribution of $X_1 + X_2$.
   (b) Compute the conditional distribution of $X_1$ given that $X_1 + X_2 = n$.

3. **Exponential (TK Problem I.5.7; Ross 1.31)**
   If $X$ and $Y$ are independent exponential random variable with respective means $1/\lambda_1$ and $1/\lambda_2$, compute the distribution of $Z = \min(X, Y)$. What is the conditional distribution of $Z$ given that $Z = X$?

4. **Borel Cantelli**
   Suppose you enter a tournament every day. The number of competitors on day $i$ is $n_i$. Assume that the probability of winning is $\frac{1}{n_i}$. Also assume that the tournament outcomes are independent.
   
   (a) If the size of the tournament is the same each day ($n_i \equiv m$), argue that you will win an infinite number of tournaments.
   (b) Now assume that the number of competitors $n_i$ grows exponentially ($n_i = e^{\alpha}$).
     Argue that you will win only a finite number of tournaments.
   (c) Find the expected number of tournaments you will win.

5. **First Step Analysis, Random walk (Ross, 1.23)**
   Consider a particle that moves along the set of integers in the following manner. If it is presently at $i$ then it next moves to $i + 1$ with probability $p$ and to $i - 1$ with probability $1 - p$. Starting at 0, let $\alpha$ denote the probability that it ever reaches 1.
(a) Argue that
\[ \alpha = p + (1 - p)\alpha^2. \]

(b) Show that
\[ \alpha = \begin{cases} 
1 & \text{if } p \geq 1/2 \\
p/(1 - p) & \text{if } p < 1/2.
\end{cases} \]

(c) Find the probability that the particle ever reaches \( n, \quad n > 0. \)

6. **Urn process (Ross, Intr. Prob. Models, Ch. 3.6.3)**

In the Polya urn process, one is given an urn with one 0-ball and one 1-ball. Draw a ball, record it \( (X_1 = 0 \text{ or } 1) \), then replace the ball and add one more like it in the urn. Draw and repeat.

(a) Show that the pmf \( p(x_1, x_2, \ldots, x_n) \) depends only upon \( \sum_{i=1}^{n} x_i \), and use it to compute this pmf.

(b) Argue that the pmf is the same as if \( X_i \sim \text{Bernoulli}(\theta) \) are independent, with the parameter \( \theta \) being a random variable of a \( U(0, 1) \) distribution.

(c) Let \( Z_n \) be the fraction of 1-balls after the \( n \)-th draw is completed and the new ball has been added. What is the distribution of the random variable \( Z_\infty = \lim_{n \to \infty} Z_n? \)

(d) (optional) Repeat (a),(c) for the urn process starting with one 0-ball and two 1-balls.