Shannon’s problem on the monotonicity of entropy

Abstract. The entropy of a random variable with density $f$ is defined as the integral of $-f \log f$. In the 1940s Shannon proved that if $X_1$ and $X_2$ are i.i.d., then the entropy of $(X_1 + X_2)/\sqrt{2}$ is at least the entropy of $X_1$. The problem whether, for i.i.d. random variables $X_i$, the entropy of $(X_1 + \ldots + X_n)/\sqrt{n}$ is an increasing sequence remained open. In this talk we will show that, indeed, entropy increases along central limit averages. The proof is based on a new variational formula which is motivated by (a proof of) the Brunn-Minkowski inequality.

Based on joint works with S. Artstein, K. Ball and F. Barthe.