Optimization and Statistical Physics: The interplay, and proofs of conjectures

Abstract. Optimization seeks to minimize a value function subject to constraints on the solution set. It also seeks to obtain efficient algorithms for finding good solutions. Statistical physics studies the evolution of systems governed by local (microscopic) rules, and tries to quantify their global (macroscopic) behavior. The Minimum Energy principle of physics states that these systems evolve towards ground states. In a sense, physicists have been studying optimization problems that occur in nature. A particular branch of statistical physics, called Spin Glass Theory, has produced a number of remarkably effective methods for hard optimization problems in Electrical Engineering and Computer Science.

This talk begins with a brief overview of the interplay between statistical physics and optimization. I will focus on the Random Assignment Problem which arises in a variety of situations of practical interest; for example, in finding minimum weight graph matchings, minimum cost paths/flows in weighted graphs, etc. By using the (non-rigorous) Replica Method of statistical physics, Mezard and Parisi computed the average value of the minimum cost assignment to be $2\pi/6$ in the “thermodynamic limit”; i.e. as the size of the system, $N$, goes to infinity. Subsequently, Parisi conjectured that when the costs are independent, rate 1, exponential random variables, the expected minimum cost for a system of size $N$ is $1/12 + 1/22 + ... + 1/2N$. This conjecture was later generalized by Coppersmith and Sorkin.

I will present a proof of the Parisi and Coppersmith-Sorkin conjectures. Subsequently, I will revisit the approach employed by the physicists and investigate some insights on the nature of optimal solutions of optimization problems. My talk will then focus attention on a recent result that proves another conjecture proposed using methods from physics on the number partitioning problem. In this problem we prove the “local REM” conjecture; equivalently, a convergence of the ordered statistics, locally, to a Poisson process and the fact that nearby configurations are uncorrelated.